STUDY OF MODEL MATCHING TECHNIQUES FOR THE DETERMINATION OF PARAMETERS IN HUMAN PILOT MODELS

First Interim Progress Report
(November 1964 through March 1965)

4380-6001-RU000

25 March 1965

Prepared under Contract NAS 1-4419
for NASA/LANGLEY RESEARCH CENTER

Prepared by: H. F. Meissinger
H. F. Meissinger
Project Manager

Approved by: R. K. Whitford
R. K. Whitford, Director
Inertial Guidance and Control Lab.

TRW Space Technology Laboratories
One Space Park, Redondo Beach, Calif.

ABSTRACT

This report presents the results obtained during the first four months of research under Contract NAS 1-4419 aimed at obtaining and evaluating human pilot response data in single-axis and multi-axis tracking tasks. This work constitutes a follow-on of earlier studies conducted under Contract NAS 1-2582 which are reported in NASA CR-143.

An extensive program of manual control experiments was carried out by two groups of three subjects each and provided a comprehensive set of tracking data, with emphasis on observing the effect of training on subject performance. The experimental design was formulated to yield representative statistical data of the ensemble of operators over a variety of tracking tasks. Evaluation of the tracking data in the next report period will complete Tasks 1 and 2 of the study program.

A series of model matching experiments was conducted using preliminary tracking data. This study resulted in the development of an improved parameter optimization criterion and of techniques for evaluation of tracking data which yield small higher order parameters. These modeling experiments, and related analysis of convergence and accuracy of the optimization process, were completed under Tasks 2 and 4 of the study. Additional analysis will be required in Tasks 5 and 6.

This interim report includes summaries of technical progress in the above areas, and a program schedule, plus an appendix of several memoranda in which detailed results and the conclusions derived therefrom are presented.

TABLE OF CONTENTS

1. INTRODUCTION

This report summarizes the results of the first phase of the research program on human pilot models performed under NASA Contract NAS 1-4419 during the four-months period ending 17 March 1965. The program has the dual objectives of (a) obtaining statistically meaningful model parameters from a large number of human pilot tracking experiments, with emphasis on two-axis studies, and (b) conducting continued analysis of model-matching techniques and their computer implementation as required.

During the report period considerable progress has been made in both directions. The major accomplishments to date are the following:

1) An extensive experimental program of single-axis and two-axis tracking studies has been formulated and executed, and data reduction is in process. Preliminary results are very encouraging.
2) Analog computer experiments have led to the development of an improved error criterion; a better understanding of the effects of initial conditions and adjustment gain on convergence problems of continuous model matching techniques; and the identification of certain "singular" cases where parameters cannot be determined.
3) Analytical investigation has shown that the precision with which system parameters can be identified is related to the relative magnitudes of their sensitivity coefficients.
4) An analytical basis for computing approximate values of the time delay (τ) and the coefficient of a third derivative term (λ) has been obtained. In addition, a modified model matching technique which avoids some of the mathematical difficulties of the continuous "approximate steepest descent" method has been developed.
5) Model matching techniques were applied to a study of display effectiveness. Statistically significant differences were

obtained between model parameters corresponding to tracking operations with different displays in cases where no differences between performance error were observed.

2. EXPERIMENTAL PROGRAM

The experimental program was formulated in such a way as to yield comprehensive data for Tasks 1, 2, and 4 of the contract work statement (Reference 1). A detailed presentation of the experimental design is given in Appendix 1.

The experiments include a total of 672 tracking runs of 2-minute duration, using either one-axis or two-axis control. A total of six subjects were used. The basic experimental variables (in addition to one- vs. two-axis tracking) were the input spectrum, the break frequency, and the controlled element (plant) time constant. In this way data were obtained for testing the effects of

a) task difficulty
b) input spectrum
c) controlled element
d) level of training

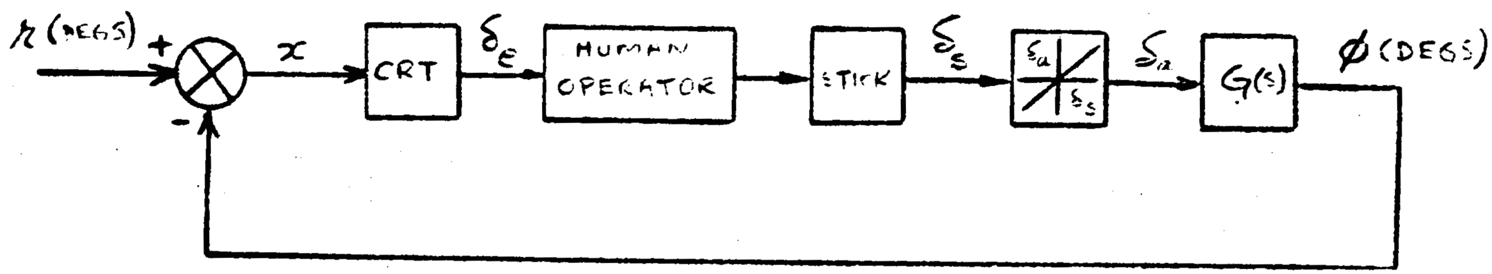
upon the parameter values in mathematical models.

All data connected with these tracking runs have been recorded on magnetic tape. The tracking error and pilot output will be used to obtain the parameters of a differential equation model (or an equivalent describing function) of the pilot's dynamic response. Model matching by continuous parameter adjustment will be employed on the basis of the methods developed under contract NAS 1-2582 (see Reference 2).

2.1 Preliminary Training Results

Training runs were completed on the single axis compensatory tracking task illustrated in Figure 1. The plant dynamics and the input signal were varied so as to simulate four tasks of increasing difficulty where task difficulty was measured in terms of the magnitude of the mean squared tracking error, $\overline{x^2}$. The tasks were defined as follows:

Task	1	2	3	4
$f_b \left(\frac{\text{rad}}{\text{sec}}\right)$	0.2	0.2	1.0	1.0
T (sec)	0.3	3.0	0.3	3.0



$$G(s) = \frac{K}{s(Ts+1)} \qquad K = 5.15 \ \frac{\text{roll rads/sec}}{\text{aileron rad}}$$

r = input roll angular disturbance (roll degrees)
x = tracking error (roll degrees)
ϕ = roll angle (roll degrees)
δ_e = visual angle (eye degrees)
δ_s = angular stick deflection (stick degrees)
δ_a = angular aileron deflection (aileron degrees)

Figure 1. Single Axis Tracking System

where f_b is the break frequency of the third order filter used to generate the input signal r(t) from a Gaussian noise generator; T is the time constant of the controlled element.

The significance of the level of training is illustrated by one subject's tracking performance shown in Figure 2 in which the mean square of the tracking error is plotted as a function of the number of training runs completed. The effect of training was found to be most noticeable in Task 4 which was considered the most difficult of the series. In all tasks the subject's tracking error decreased with the progress of training.

Similar data were obtained for all six subjects and will be included in a subsequent progress report when the effects of training on model parameters have been evaluated.

An analysis of variance will be applied to all experimental data to determine the statistical significance level.

3. DESCRIBING FUNCTION MODELS OF THE HUMAN PILOT

In order to provide a basis for comparison with previously published models and to provide a standard against which the models of the new experimental series can be compared, selected runs will be processed through the STL spectral analysis program. A detailed description of the program is given in Appendix 2.

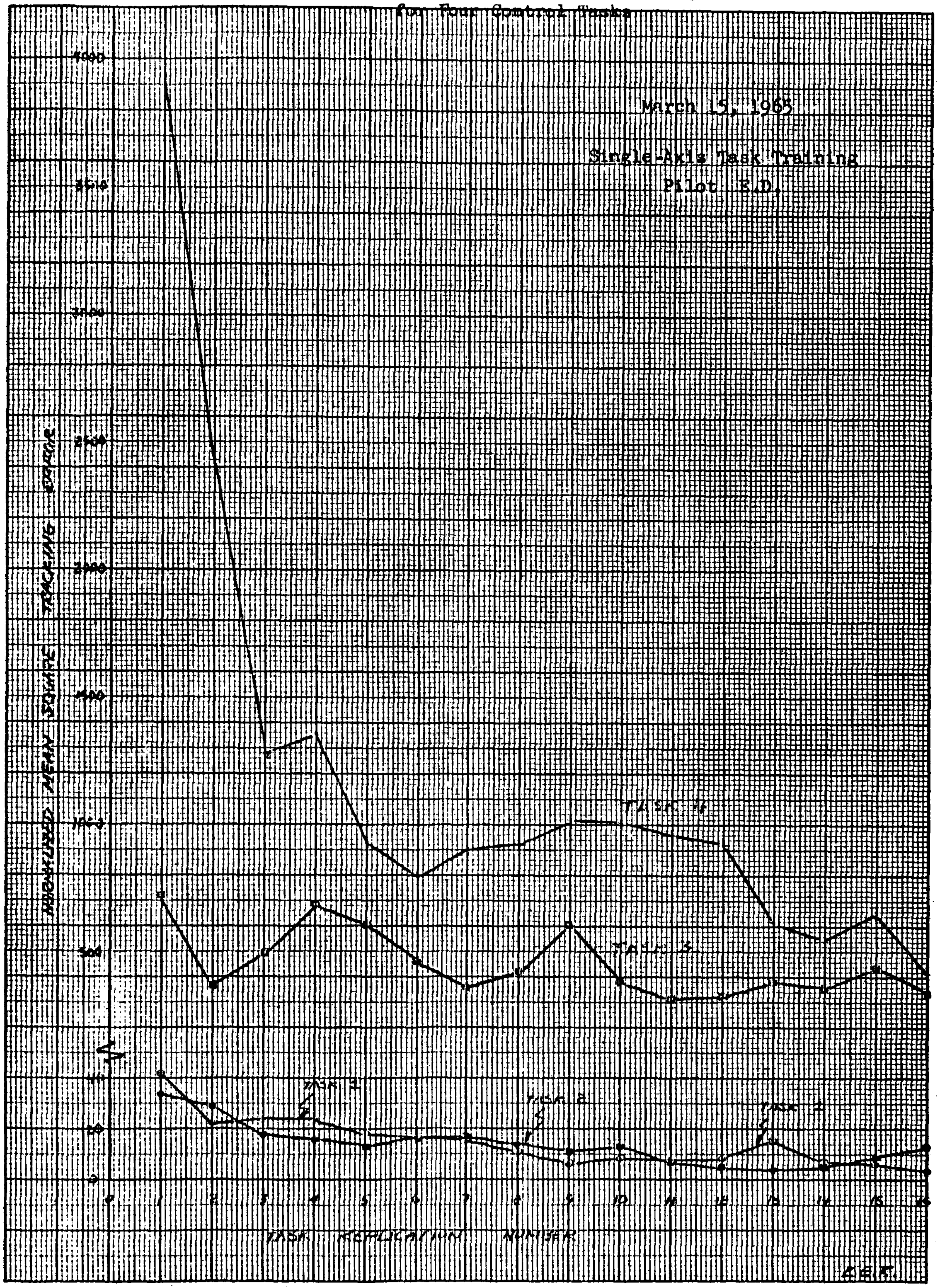
Basically, the technique consists of estimating a describing function for the pilot from the relationship

$$Y_p(j\omega) = \frac{S_{ry}(j\omega)}{S_{rx}(j\omega)} \qquad (1)$$

where $S_{ry}(j\omega)$ is the cross spectral density between reference input and pilot output and $S_{rx}(j\omega)$ is the cross spectral density between reference input and tracking error.

In order to obtain the ratio (1), analog records of the reference input r(t), tracking error x(t), and pilot output y(t) are digitized and used as input data to the spectral program. The digital

Figure 2. Results of Operator Training for Four Control Tasks



computer first evaluates the cross-correlation functions $\phi_{rx}(t)$ and $\phi_{ry}(t)$, and then obtains the cross spectra by calculating the Fourier transform of the correlation functions. As a result of two passes through the program the cross spectra are obtained in the form of amplitude $|S(j\omega)|$ abd phase, arg $S(j\omega)$, as function of frequency. Then, the amplitude and phase of the pilot's describing function are obtained from

$$|Y_p(j\omega)| = \frac{|S_{ry}\ (j\omega)|}{|S_{rx}\ (j\omega)|} \tag{2}$$

$$\arg Y_p(j\omega) = \arg S_{ry}(j\omega) - \arg S_{rx}(j\omega) \tag{3}$$

The resulting values of amplitude and phase will be plotted vs. frequency, and analytic functions of the form

$$Y_p'(j\omega) = \frac{K_o e^{-\tau s}\ (1 + jT_1\omega)}{(1 + jT_2\omega)\ (1 + jT_3\omega)} \tag{4}$$

will be fitted to this data by adjustment of the parameters K_o, τ, T_1, T_2, and T_3.

A detailed discussion of run length, sampling frequency, and confidence limits for the spectral estimates is given in Appendix 2.

4. PRELIMINARY MODELING EXPERIMENTS ON HUMAN TRACKING DATA

Model matching experiments were conducted on preliminary human operator tracking data to explore influence of display characteristics on tracking performance, and to investigate and improve the parameter optimization technique in cases where difficulties in the rate of adjustment, convergence properties and determinacy were encountered. A detailed report and analysis is contained in Appendix 3. The human tracking data used here came from an experimental series which has been described and evaluated in Appendix 8.

In summary, the most significant results derived from this study are the following:

a) For rapid convergence and improved definition of optimum model parameters a modified error criterion $f_2(\epsilon, \dot{\epsilon})$ was developed which is more sensitive to parameter variation from the optimum point without causing stability problems when ϵ, $\dot{\epsilon}$ are large. The criterion function has limited slope, being essentially an absolute value criterion for large ϵ, $\dot{\epsilon}$.

b) The behavior of the model matcher was studied under conditions where small parameters occurred in the highest order term of the model equation. Simplified first order models were compared with second order models in terms of parameter definition and model matching accuracy.

c) Uncontrolled drift in the model parameters (observed in some cases even with the improved error criterion function) was traced to critical combinations of system parameters and the use of very low frequencies of excitation. Means of detection and possible resolution of such indeterminacies were explored.

The results of the study required further experimental research on parameter convergence (see Section 5) and additional analysis which will be discussed under Section 6 and in Appendices 5 and 7.

5. CONVERGENCE STUDY OF FIRST-ORDER MODEL PARAMETERS

In order to clarify the effect of adjustment gain and parameter initial conditions on the convergence properties of the continuous model matching technique, an extensive experimental study was conducted. The detailed results of this study are reported in Appendix 4. The pilot's response was assumed to be represented by the first-order equation (synthetic pilot)

$$\dot{y} + b_1 y = b_2 \dot{x} + b_3 x \tag{5}$$

The corresponding model equation is

$$\dot{z} + \beta_1 z = \beta_2 \dot{x} + \beta_3 x \tag{6}$$

The continuous model matching technique was used to adjust model parameters in order to minimize the criterion function

$$f = (e + q\dot{e})^2 = [\ (z-y) + q\ (\dot{z}-\dot{y})\ ]^2 \tag{7}$$

where e is the matching (or output) error. A fixed value of $q = 0.05$ was used in this study.

The tracking error from a 2-minute compensatory tracking run was used as the input signal $x(t)$ for both the system (5) and model (6). The system parameters b_1, b_2, and b_3 were held fixed. The major conclusions from the experiment were:

a) Repeatability of parameter adjustment could be obtained only for certain initial conditions.

b) Long term (1 and 2 minute) convergence of the β_i to the corresponding b_i can be obtained with an average accuracy of $\pm$ 6 percent. If the transfer function corresponding to (5) is written, the new parameters (gain and time constants) can be determined to an average accuracy of $\pm$ 4 percent.

c) For short-term (less than 15 sec) convergence, accuracies of $\pm$ 10 percent could be obtained using the largest possible adjustment gain.

d) The model-matching technique can be used to detect control reversals, but more research will be required before the accuracy of such detections can be ascertained.

e) More analytical work is required to obtain general results on the convergence of model parameters to desired values, even with simple first-order models.

6. ANALYTICAL RESULTS

6.1 Relative Sensitivity of Model Parameters

In order to obtain a clearer understanding of some of the convergence problems outlined under Sections 4 and 5 and in Appendices 3 and 4, additional analytical study was desirable. It was noted above that convergence of transfer function parameters to correct values was better than that of differential equation parameters. An analysis of the corresponding sensitivity factors and their relative magnitude was made to explain this result on a mathematical basis (see Appendix 5).

It has been shown previously (see Reference 2) that those parameters having the largest relative sensitivity will be defined with the greatest precision. The present study established the relative magnitude of the sensitivity coefficients

$$u_i = \frac{\partial z}{\partial \beta_i}, \quad i = 1, 2, 3$$

and

$$v_0 = \frac{\partial z}{\partial K_0}, \quad v_1 = \frac{\partial z}{\partial \tau_1}, \quad v_2 = \frac{\partial z}{\partial \tau_2}$$

where K_0, τ_1, and τ_2 are parameters in the transfer function corresponding to the differential equation (6). Analytical expressions for the sensitivities u_i, v_i and their frequency dependence indicate that the v_i (transfer function parameter sensitivities) are at least an order of magnitude larger than the u_i (differential equation parameter sensitivities). This explains the more accurate definition of the transfer function parameters, particularly of K_0. However, since implementation of the transfer function format requires more computational equipment than the differential equation form, as well as being limited to linear models, additional study of this problem is indicated. Wherever feasible, the transfer function form will be used in the remainder of the program.

6.2 Approximate Identification of a Missing Time-Delay Term

This study, reported in detail in Appendix 6, was undertaken to show the feasibility of evaluating a missing reaction time term in a mathematical model of the form

$$\ddot{z} + \alpha_1\dot{z} + \alpha_2 z = \alpha_3 x(t - \tau) \tag{8}$$

without using the special computing equipment that would be required to generate time-delay. It was assumed that the α-parameters had been previously obtained, so that only the τ parameter needs to be computed.

The major conclusions of the analysis are:

a) A missing time delay term τ can indeed be computed approximately without using actual time-delay equipment, by using linear extrapolation near the solution for $\tau = 0$.

b) An error analysis based on sinusoidal excitation signals shows that for time delays of the order of magnitude expected in compensatory tracking studies (e.g. $\tau \cong 0.15$ sec) and input frequencies below about 3 rad/sec, τ can be determined with an accuracy better than 10 percent. In view of the simplicity of implementation, this is a significant result. Computer studies in Task 2 will be based on these results.

6.3 Approximate Identification of Higher Order Terms

The effect of small higher order terms not included in the formulation of the model equation was analyzed, and methods for approximating the parameters associated with such terms were investigated in detail. This work is reported in Appendix 7. Similar in scope to the study discussed in Section 6.2 (Appendix 6) the objective of this analysis was to provide the means for upgrading model matching accuracy without reprogramming the basic model equation.

The method consists of a linear extrapolation of the model output variable z in the vicinity of the case $\lambda = 0$. For example, if the original model equation programmed on the computer is

$$\dot{z}_0 + \beta_1 z_0 = \beta_2 x(t) \tag{9}$$

the solution z_0 can be extrapolated to approximate the solution of

$$\lambda \ddot{z} + \dot{z} + \beta_1 z = \beta_2 x(t) \tag{10}$$

for non-zero λ. The extrapolation is performed in terms of the first order sensitivity $u_\lambda = \partial z_0/\partial\lambda$, namely

$$z \simeq z_0 + u_\lambda \lambda \tag{11}$$

for sufficiently small values of λ. The method can be readily extended to model equations of higher order. A mathematical difficulty due to the change of the order of the model equation for $\lambda = 0$ and $\lambda \neq 0$ exists at time $t = 0$. However, the singularity of the initial response $z(0)$ and of its sensitivity u_λ occurring at $\lambda = 0$ is of no practical concern in the modeling procedure since, in general, the initial conditions have only a transitory influence on model output.

This study has provided the following major results (see Appendix 7):

1. For practical purposes the extrapolation (11) is useful for reasonably small parameter values λ and excitation frequencies below 5 to 10 rad/sec.
2. The missing parameter λ can be approximated with accuracies on the order of 10 percent or better under conditions representative of human pilot dynamic responses. The approximation accuracy is comparable to the results obtainable for a missing time delay term (see Section 6.2).

3. Computer programs have been devised that include the higher order term but avoid singularity of the computer solution for the case $\lambda = 0$.

6.4 Modified Parameter Optimization Strategy

The continuous model matching technique being used at the present time suffers from a lack of mathematical precision in the definition of the gradient components. Appendix 8 describes a modified strategy which avoids this difficulty by maintaining constant model parameters and using an open-loop linear extrapolation to compute parameter increments. It is shown that not only is this method mathematically rigorous, but it shows promise of resulting in an improved rate of convergence and fewer stability problems.

The method is primarily useful in the vicinity of the correct parameter values, where linear extrapolation produces valid results. If initial conditions of model parameters are far from their correct values, several iterative steps may be required to produce convergence. The new method has been tested with a known system, but has not been applied to parameter identification in human operator models. It is planned to apply this method during future phases of the program.

7. EVALUATION OF DISPLAY SENSITIVITY BY HUMAN OPERATOR MODELS

In order to further test the practical application of human operator models, a preliminary study was performed to examine the effect of display characteristics upon the model parameters. The hypothesis being tested was that changes in display characteristics would affect the model parameters significantly, even in situations where performance (as measured by rms tracking error) was not affected significantly. The detailed description of the experiment is given in Appendix 9.

Two basic experimental conditions were used: (1) Viewing a CRT display through an eyepiece with a 60° field of view such that an error signal representing 1° of vehicle motion in pitch subtends a 1° visual angle at the eye (use of horizon as pitch reference); (2) Viewing the CRT with the naked eye at a distance of 28 inches (use of artificial horizon). No significant difference was found between rms tracking errors obtained for the two displays. The data was then analyzed using a first order mathematical model. Significant differences in model gain (at the 10 percent level of significance) were observed between the two experimental situations. With the less sensitive display the operator lowered his gain, indicating a lower crossover frequency and consequently a more difficult task. A detailed statistical analysis is given in Appendix 9.

These results are particularly encouraging since the objectives of the present research program include application of model matching to tasks related to flight control. This study shows the feasibility of using human pilot models to provide a quantitative basis for evaluation of proposed displays.

8. PROGRAM SCHEDULE

The program was initiated with tracking experiments and analytical studies required under Tasks 1, 2, and 4 of the contract work statement. Work under Task 4 was performed ahead of Task 3 since the analytical and experimental results obtainable from Task 4 are needed to complete Tasks 1 and 2.

A detailed work program and schedule of milestones is presented on pages 14a and 14b. In the next report period Tasks 1, 2, and 4 will be completed and the experimental program of Task 3 (two-axis control with cross-coupling) will be initiated. Manpower utilization will continue at a uniform rate of 2.3 MTS per week, with laboratory and technical support added during periods of increased computational and data reduction activity.

No major technical problems, manpower problems, or problems in facility utilization are foreseen. Expenditures are expected to continue in accordance with the budget forecast transmitted to NASA in January 1965.

CONTRACT TIME SCHEDULE

NAS1-4419

March 22 - November 8

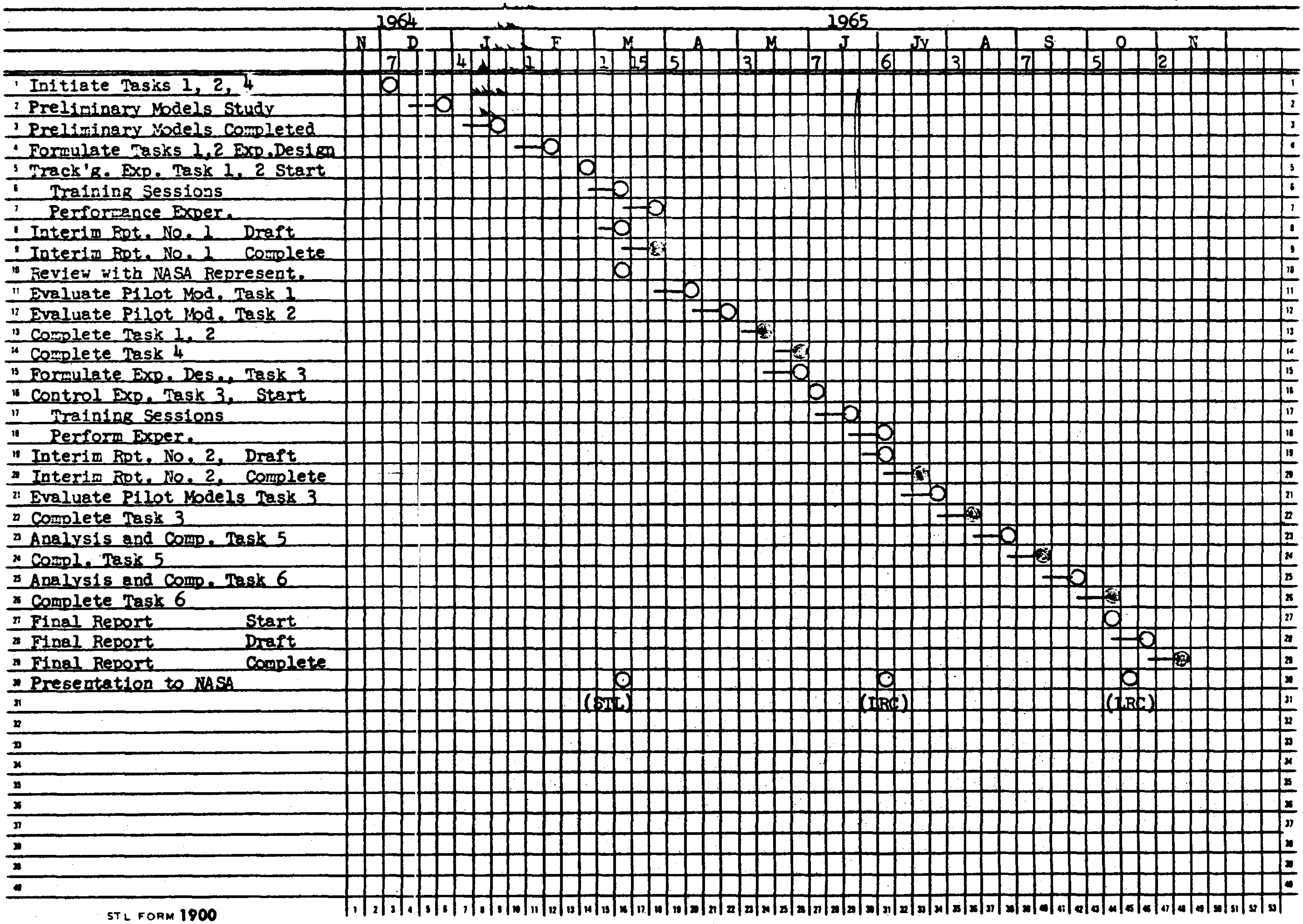
JOB NO.	START DATE	DESCRIPTION	TIME ALLOT'MT (weeks)	TIME DISTRIBUTION Analysis	Computer	Report
1	March 22	Task 1	3	1.5	1.5	
2	Apr. 12	Task 2	2		2	
3	Apr. 26	Complete Tasks 1 & 2	2	2		
4	May 10	(Complete Task 4) (Expt. Design B)	4	2 1*	2	
5	June 7	Set up Expt. B	1		1 (AD)	
6	June 14	Training runs	2		2 (AD)	
7	June 28	(Performance runs) (Report 2)	2		2 (AD)	2*
8	July 12	(Complete Report 2) (Start Task 3)	2		1	1
9	July 26	Complete Task 3	3	2*	2	
10	Aug. 16	Task 5	4	2	2	
11	Sept. 13	Task 6	4	2	2	
12	Oct. 11	Final Report	4			4
		TOTAL	33	9.5*	12.5	7*

<u>Note:</u>

* denotesthat only one MTS will be allotted this time.
All other figures require the participation of two MTS.

HUMAN PILOT MODELS
NAS 1-4419

Milestones



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REFERENCES

1. "Mathematical Models of Human Pilot Responses," Proposal 3667.000, dated 11 June 1964 (submitted to NASA/Langley Research Center).

2. "A Study of Model Matching Techniques for the Determination of Parameters in Human Pilot Models," by G. A. Bekey, H. F. Meissinger, and R. E. Rose. STL No. 8426-6006-RU000, dated 2 May 1964. (Published under NASA Contractor Series CR-143, January 1965.)

APPENDICES

1. "Experimental Design for Tasks 1, 2, and 4 of NASA Contract NAS 1-4419," No. 9352.2-7, by E. P. Todosiev, dated 12 February 1965.

2. "Procedure for Digital Computation of Power Spectra of Analog Records," No. 9350.4-1 , by G. A. Bekey, dated 18 February 1965.

3. "Preliminary Modeling Experiments on Human Tracking Data," by H. F. Meissinger, R. E. Rose, and E. P. Todosiev, No. 9350.6-157, dated 29 March 1965.

4. "Convergence Study of First Order Model Parameters," by E. P. Todosiev, No. 9352.2-16, dated 25 March 1965.

5. "Relative Sensitivity of Human Pilot Model Parameters," by H. F. Meissinger, No. 9350.6-153, dated 26 February 1965.

6. "Analysis of Time Delay Approximation in Human Pilot Models," by H. F. Meissinger, No. 9350.6-152, dated 26 February 1965.

7. "Approximation of Small Parameters Associated with Higher Order Terms in Human Pilot Models," by H. F. Meissinger, No. 9350.6-156, dated 18 March 1965.

8. "Modified Parameter Optimization Strategy Using Exact Gradient Components," by H. F. Meissinger, No. 9350.6-155, dated 3 March 1965.

9. "Evaluation of Display Sensitivity by Human Operator Models," by L. G. Summers, No. 9352.2-5, dated 9 March 1965.

SPACE TECHNOLOGY LABORATORIES, INC.

a subsidiary of Thompson Ramo Wooldridge Inc.

INTEROFFICE CORRESPONDENCE

9352.2-7

TO: H. F. Meissinger

CC:

DATE: 12 February 1965

SUBJECT: Experimental Design for Tasks 1, 2, and 4 of NASA Contract NAS 1-4419

FROM: E. P. Todosiev

BLDG.	ROOM	EXT.
R2	1186	12250

The experimental study section of the contract work statement will be performed using two experimental designs. Tasks 1, 2, and 4 will be covered by the design described below while Tasks 3, 5, and 6 will require a more complex experimental design which will be formulated later. Only one experimental design is required for Tasks 1, 2, and 4 as these tasks are concerned with the dependence of human parameters on task difficulty when the human operator is performing either a single-axis or dual-axis tracking task.

Training and performance experiments will be performed on two simulated tracking systems to obtain experimental data for Tasks 1, 2, and 4. Both experiments will be concerned with compensatory tracking of a spot on a CRT display where one system is restricted to single-axis control and the other to two-axis control with symmetrical uncoupled plant dynamics. Two alternate plant dynamics were chosen from Creer, et al (Reference 1) to give satisfactory and unsatisfactory performance respectively. Input signals to the systems will be obtained by passing gaussian noise through a third order filter to obtain a spectrum similar to that used by Elkind (Reference 2). The input power to the systems will be held constant at 1" RMS deflection on the CRT display. Experimental runs will be of 3 minutes duration and only 2 minutes will be scored. The experiment is divided into two major portions: the training experiment, and the performance experiment.

I. TRAINING EXPERIMENT

A) System Configurations

- o Single-axis control
- o Dual-axis control

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B) Range of Variable Values

1. Input spectrum break frequency (f_b)
   - 0.2 rad/sec
   - 1.0 rad/sec

2. Plant Dynamics

$$G(s) = \frac{\phi}{\delta_a}(s) = \frac{K}{s(Ts + 1)}$$

K $\left(\frac{\text{roll rads/sec}}{\text{aileron rad}}\right)$	T (sec)
5.15	0.3
5.15	3.0

where

$K = TL_{\delta_a} \left(\frac{\text{roll rad/sec}}{\text{aileron rad}}\right)$

L_{δ_a} = roll angular acceleration per unit aileron angular deflection $\left(\frac{\text{roll rad/sec}^2}{\text{aileron rad}}\right)$

δ_a = aileron angular deflection (aileron rad)

ϕ = roll angle (roll rads)

δ_s = stick angular deflection (stick rads)

δ_s max. = 20 degrees

The operating gains were chosen from Figure 11 of Reference 1 under the assumption that $\delta_s = \delta_a$.

3. Subjects

Single-axis	Dual-axis
3 subjects (Group 1)	3 subjects (Group 2)

4. Run characteristics

Sessions	5
Replications per session	4

Only the fourth replication of each session will be recorded. A full factorial design will be used and the order of the four experimental variables will be randomized for each subject and each session.

C) Summary

System configurations (SC)	2
Plant dynamics (g)	2
Filter break frequency (f_b)	2
Subjects (S)	3
Replication (R)	20

Total runs = SC x G x f_b x S x R = 480

Total Runs Recorded = $\frac{480}{4}$ = 120

II. PERFORMANCE EXPERIMENT

The performance experiment will be performed with the same subjects as in the training experiment and will be conducted as an extension of the training experiment. All variable values will remain the same with the sole exception of the run characteristics which will consist of only 2 sessions with 4 replications per session. All runs will be recorded.

Summary

SC = 2
G = 2
f_b = 2
S = 3
R = 8

Total Runs = SC x G x f_b x S x R = 192

Total Runs Recorded 192

III. ESTIMATED MAGNETIC TAPE AND RUNNING TIME REQUIREMENTS

Tape Requirements

Total runs to be recorded = 120 + 192 = 312
Tape speed = 3.75"/sec = 18.75 ft/min
One 2-min requires 2 x 18.75 = 37.5 ft

Allow 50 ft for each run

Total tape footage required = 50 x 312 = 15,600 ft

3600 ft per roll of tape

Number of rolls required = $\frac{15,600}{3600}$ = 5 rolls

Time Requirements

Total training runs =	480
Total performance runs	192
Total runs required	672

Time per run = 5 min

Total running time = $\frac{5 \times 672}{60}$ = 56 hrs

Total down time = 100% = 56 hrs

Actual time required = 112 hrs = 2.8 weeks

IV. UTILIZATION OF EXPERIMENTAL DATA

The experimental data will be required in Tasks 1, 2 and 4 as follows:

Task 1: Effect of Task Difficulty on Human Tracking Performance

Analysis of the experimental data with the model-matcher will yield the following results:

1) Scaling of task difficulty with respect to input signal bandwidth and variable plant dynamics where the mean square tracking error is used as the criterion
2) Effect of training on the parameters of the differential equation form of the human pilot model,

$$\ddot{z} + \alpha_1 \dot{z} + \alpha_2 z = \alpha_3 \dot{x} + \alpha_4 x$$

3) Effect of input spectrum and plant dynamics on the power match between the model and the human operator output.

An analysis of variance will be applied to all experimental data to determine its statistical significance level.

Task 2: Effect of Higher Order Models on Matching Error

In this task advanced model forms will be evaluated. Specifically, these models will have the forms:

$$\ddot{z}(t) + \alpha_1 \dot{z}(t) = \alpha_3 \dot{x}(\tau) + \alpha_4 x(\tau) \quad \text{and}$$

$$\lambda \dddot{z} + \ddot{z} + \alpha_1 \dot{z} + \alpha_2 z = \alpha_3 \dot{x} + \alpha_4 x$$

where $x(\tau) = x(t - \tau)$

Evaluation of the experimental data will yield the following results:

1) Effect of a time transport lag on the model matching error.
2) Effect of a third order term on the model matching error.

Methods of computation of the time delay τ and the third order term coefficient λ will be studied.

Task 4: Advanced Model Matching Methods

Analytical studies will be made of the iterative closed-loop model matcher as well as the open-loop model matcher. The model matchers developed will be used on the experimental data. The results of Task 4 can serve as a check on the results of Tasks 1 and 2 as these parameters will have been already determined.

References

1. Creer, B. Y., et al, "A Pilot Opinion Study of Lateral Control Requirements for Fighter-Type Aircraft," Ames Research Center, Moffett Field, California. NASA Memo 1-29-59A.

2. Elkind, E. I., "Characteristics of Simple Manual Control Systems," MIT Lincoln Laboratory Report No. 111, 6 April 1956.

INTEROFFICE CORRESPONDENCE 9350.4-1

TO: H. F. Meissinger CC: Distribution DATE: 18 February 1965

SUBJECT: Procedure for Digital Computation of Power Spectra of Analog Records

FROM: G.A. Bekey
BLDG. R2 ROOM 1154 EXT. 12300

Computation of power spectra of continuous data is performed by CDRC following a sequence of the following operations:

(1) Analog data recorded on FM tape must be digitized.

(2) The digital tape must be converted into a 7094 compatible format.

(3) The IBM 7094 correlation and spectral analysis program[1] must be run.

This memorandum outlines the major aspects of the three steps listed above, with reference to computation of spectra of manual tracking records.

1. Digitizing of Analog Data

If the analog data are recorded at 3-3/4 in. per second, it is recommended that they be converted to digital form as follows:

(a) Record all runs to be digitized on the same tape, using five channels of the tape.

(b) Digitize at 125 samples/sec, commutated among the four functions. The net sampling frequency becomes

$$f_s = \frac{125}{5} = 25 \text{ samples/ second} \quad (1)$$

This sampling rate is sufficiently high to avoid Nyquist folding if the data are filtered prior to sampling with a filter cut-off frequency.

$$f_{co} = f_s/2 = 12.5 \text{ cps} \quad (2)$$

2. Decommutation

The sampled data points from the A/D converter may not have sequential location, and must be decommutated and separated into labeled records. The program for performing this process is known as the "FM-FM Processing Program."

3. Selection of Parameters for Spectral Program

The program requires the selection of the following quantities:

Δt = the interval at which the correlation functions are computed.

n = the number of data points to be used in the computation.

m = number of lag values used in the computation of the correlation function

The significance of these numbers can be related to the definition of the auto correlation function $\phi_{xx}(\tau)$.

$$\phi_{xx}(\tau) = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{i=-N}^{N} x(t_i)\, x\,(t_i + \tau) \tag{3}$$

Since the data are available only at discrete points, where

$$t_i = i\, \Delta\, t$$

The lag values τ must also be separated by integral multiples of Δt, and (3) becomes

$$\phi_{xx}(k\Delta t) \cong \frac{1}{n} \sum_{i=0}^{n} x(i\Delta t)\, x\,(i\Delta t + k\Delta t) \tag{4}$$

The number of lag values m is equal to the number of values of k in Eq. (4) which are used in the computation;i.e. k = 1, 2, 3....m.

For the sampling frequency suggested in Eq. (1), we have

$$\Delta t = \frac{1}{25 \text{ samples/sec}} = 0.04 \text{ seconds} \tag{5}$$

The number of lag values, m, determines the frequency resolution Δf possible. Assume that m = 250 is used. Then

$$\Delta f = \frac{1}{2(\Delta t)m} = \frac{1}{2(.04)(250)} = 0.05 \text{ cps} \tag{6}$$

If this resolution is too fine, fewer lag values can be used. As noted below, the number of lags is directly related to the cost of computation.

If four minutes of data are used, the number of data points becomes

$$n = 4 \text{ min} \times 60 \frac{\text{sec}}{\text{min}} \times 25 \frac{\text{samples}}{\text{sec}} = 6000 \tag{7}$$

4. <u>Confidence Band</u>

From the fact that spectral estimates are distributed as chi-square, Blackman and Tukey[2] give confidence limits for various measurement techniques. For the method used in this program, the 90% confidence band of the computed spectral density is, approximately,

$$90\% \text{ conf. band} \cong 20 \sqrt{\frac{m}{2n-m}} \quad \text{db} \tag{8}$$

For $n = 6{,}000$

$m = 250$

$$90\% \text{ conf. band} \cong 20 \sqrt{\frac{250}{12{,}000-250}} = 2.9 \text{ db} \tag{9}$$

i.e. there is a 10% chance that the true value of spectral density $\hat{S}_{ff}(\omega)$ would be outside of the band

$$(S_{ff}(\omega) - 1.45 \text{ db}) < \hat{S}_{ff}(\omega) < (S_{ff}(\omega) + 1.45 \text{ db}) \tag{10}$$

If only 5,000 points are used, the confidence band becomes 3.2 db. Either 5,000 or 6,000 points would be adequate. It can be noted the width of the confidence band is approximately inversely related to the frequency resolution desired - i.e. the finer the resolution the broader are the confidence limits.

5. <u>Outputs</u>

The digital program can provide an input tape for the digital plotter, and from any pair of records x(t) and y(t), the following can be obtained:

(a) Autocorrelation functions $R_{xx}(\tau)$ and $R_{yy}(\tau)$

(b) Power spectral densities $S_{xx}(f)$ and $S_{yy}(f)$

(c) Cross correlation function $R_{xy}(\tau)$ and cross spectral density $S_{xy}(f)$. Since $S_{xy}(f)$ is a complex number, the program provides the magnitude and phase which can be plotted separately.

(d) The coherence function, C(f) is also computed, thus giving a measure of the degree of linearity in the relationship between x(t) and y(t);

$$C(f) = \frac{|S_{xy}(f)|^2}{S_{xx}(f)S_{yy}(f)} \tag{11}$$

7. Preparation of Load Sheets

A typical pair of load sheets, using the numbers computed above, is attached. The following additional comments can be made:

(a) Normalization refers to dividing by the mean square value, so that the maximum value of the correlation function becomes 1.

(b) Tape number, file number, etc. are provided by CDRC.

(c) The program can begin with an arbitrary point. For example, if each record contains five minutes or 7,500 points at 25 pts/sec, it is possible to start at the 1000th point and use the succeeding 5000 points, as indicated in the load sheet.

(d) If less resolution is required, every j-th point can be used.

(e) When both x and y functions are present and autocorrelation is requested, the program computes both autocorrelations. Where cross-correlation is requested, the program computes the coherence function as well.

An important point to note is that an option in the program calls for removal of the mean from a record (if there is a non-zero mean).

6. Preparation of Input Functions

The following arrangement of data is suggested for each record:

Channel 1	Reference input $r(t)$
2	Tracking error $x(t)$
3	Pilot output $y(t)$
4	Model output $z(t)$
5	Matching error $e(t)$
6	Blank
7	Voice track for run identification

The synthesis of human operator models is based on the relationship:

$$Y_p(j\omega) = \frac{S_{ry}(j\omega)}{S_{rx}(j\omega)} \qquad (12)$$

where $Y_p(j\omega)$ is the pilot "describing function" and $S_{ry}(j\omega)$ and $s_{rx}(j\omega)$ are obtained from two passes through the spectral program.

8. Estimation of Running Time

Reference 1 gives the following estimate for IBM 7094 running time to:

$$t_c \cong 2 \times 10^{-6} \left\{(60\ m + 100)\ n\ X + \left[(25X + 100)\ m + 1000\right]P\right\} \text{seconds} \qquad (13)$$

where n and m have been defined previously

P = number of power spectra output points

X = 1 for 1 function (autocorrelation only)

2 for 2 function (autocorrelation only)

3 for 2 functions when cross correlation is requested.

For large values of m (say $m > 100$), and $X = 3$, Eq. (13) can be simplified to

$$t_c \cong 3.6 \times 10^{-4}\ mn \text{ seconds}$$

For $m = 250$ and $n = 5000$, we obtain $t_c \cong 450$ sec = 7.5 min
With computer charges of $420/hr, this represents approximately $55.00 per spectrum, not counting data reduction, digitizing and plotting.

G. A. Bekey

GAB:jbk
Attachment

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PAGE 1 OF 2

NAME [illegible]

PRIORITY

5007.0

PROBLEM NO.

KEYPUNCHED BY

NO. OF CARDS

VERIFIED BY

KEYPUNCH			DO NOT KEYPUNCH
(LAG) =	250	B	Number of lags to compute.
(N) =	5000	B	Number of points to process.
(DELT) =	0.04		Delta time (1/sampling rate)
(DELF) =	005		Frequency resolution
(MEAN) =	1	B	1 = remove the mean from the data 0 = do not remove
(ACC) =	0	B REL	1 = calculate autocorrelation and power spectra on this function. 0 = do not
(CCC) =	1	B	1 = calculate crosscorrelation and cross spectra on this function. 0 = do not.
(ANØRM) =	1	B	0 = normalize autocorrelation, 1 = do not.
(CNØRM) =	1	B	0 = normalize crosscorrelation, 1 = do not.
(XTAPE) =		B	Tape unit number for the X tape.
(TWNØ) =		B	Time word number } CHOOSE ONE
(TWNØ)H1 =		+0 REL	DAD or DACV for DAD or DACV format tapes } CHOOSE ONE
(XID) =		B	X data word number in frame } CHOOSE ONE
(XID)H1 =		+0	X data word name for DAD or DACV format tapes } CHOOSE ONE
(DENS) =		B	1 = high density 0 = low density
(FILENØ) =		B	File number
(FSIZE) =		B	Number of words per frame } IGNORE FOR DAD or DACV
(SKIP) =		B	Number of I.D. records to skip } IGNORE FOR DAD or DACV
(START) =	40		Start time
(J) =	1	B REL	Use every Jth point.

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DATE ______ PAGE 2 OF 2

NAME ______ PRIORITY ______

PROBLEM NO. ______ KEYPUNCHED BY ______

5007.0

NO. OF CARDS ______ VERIFIED BY ______

KEYPUNCH		DO NOT KEYPUNCH
(YTAPE) = ______B		Tape unit number for the Y tape.
(YID) = ______B		Y data word number in frame } CHOOSE ONE
(YID)H1 = ______$\overset{+}{0}$	REL	Y data word name for DAD or DACV format tapes.
		If YTAPE = XTAPE the following may be omitted.
(YDENS) = ______B		Y tape density 1 = high 0 = low
(YTWNØ) = ______B		Y tape time word number } CHOOSE ONE
(YTWNØ)H1 = ______$\overset{+}{0}$		DAD or DACV
(YFILE) = ______B		Y tape file number
(YFSIZE) = ______B		Y tape number of words per frame } IGNORE FOR DAD OR DACV
(YSKIP) = ______B		Y tape number of I.D. records to skip } IGNORE FOR DAD OR DACV
(YSTART) = ______		Y tape start time
(YJ) = ______B	REL	Y tape process every Jth point
$ REL		END OF CASE
$ REL		END OF RUN

EB

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INTEROFFICE CORRESPONDENCE

9350.6-157

TO: Distribution CC: DATE: 29 March 1965

SUBJECT: Preliminary Modeling Experiments on Human Tracking Data

FROM: H. F. Meissinger HFM
R. E. Rose
E. P. Todosiev

BLDG. R2 ROOM 1086 EXT. 22115

1. INTRODUCTION

This report describes preliminary model matching experiments conducted with single-axis human operator tracking data. The tracking task was performed with a controlled element of second order and was intended to demonstrate the effect of various display characteristics on the human operator's model. The objective was to employ the model matching procedures developed under Contract NAS 1-2582 (see Reference 1) in a practical control task of the type to be further investigated under the present contract (Tasks 1, 2, and 3).

In the course of these experiments it was found that under the prevailing low frequencies of the perturbation signal (filtered random noise with break frequency $\omega_b = 0.33$ rad/sec) the parameters did not always converge satisfactorily to fixed steady state values. In some cases non-unique sets of parameters were obtained. These observations made it necessary to examine the stability and convergence characteristics of the model matching process more carefully and to develop an improved optimization criterion. The experiments were extended to the case of a system with known parameters ("synthetic pilot") to calibrate the accuracy of parameter matching and to detect the causes of parameter indeterminacy.

Parameter indeterminacy was traced to cases of critical system parameter combinations where model matching is not feasible under low frequency excitation. This difficulty is of a general nature in system identification, not restricted to modeling by continuous parameter tracking.

Another systematic difficulty was noted when the parameters associated with the highest order term of the model equation assumed very small values. This case led to an investigation of parameters which change the order of the model equation, and of suitable computer programming techniques (see Reference 3). The study also provided insight into the causes of fluctuation in steady state parameters, and into essential differences in parameter sensitivities (see Reference 4).

It should be noted that the results and observations in this report are based on very limited experimental data from only few sets of experimental conditions. While both the observations and analysis indicate a broad application of the results, further experimental evidence is needed. It is hoped to provide some of this evidence during later phases of the program.

2. EXPERIMENTAL PROCEDURE

The experimental model matching studies were performed on single-axis human operator tracking data. The modeling procedure was an extension of the techniques developed previously (Reference 1). In addition to modeling the response of actual human operators, model matching was also performed on synthetic pilots with known parameters for purposes of testing the computer program, determining the accuracy of the parameter values obtained, and exploring sources of unsatisfactory model matching performance.

Tape recorded human tracking data served as experimental data for this study. In those cases where synthetic pilot parameters rather than actual human data were determined by the model matcher input signals $x(t)$ from human tracking runs were inserted to provide a representative spectrum of excitation frequencies.

The compensatory tracking task is sketched in Figure 1. The model included not only the human operator response characteristics but also the display characteristics and hand controller response. The experimental approach and the conditions under which the tracking data were obtained are described in Reference 2.

Most of the model matching operations were performed by simultaneous adjustment of all four parameters in a continuous closed-loop process. In addition, iterative and semi-iterative techniques were used in some cases in an attempt to improve convergence. The continuous and iterative techniques have been fully described in Reference 1.

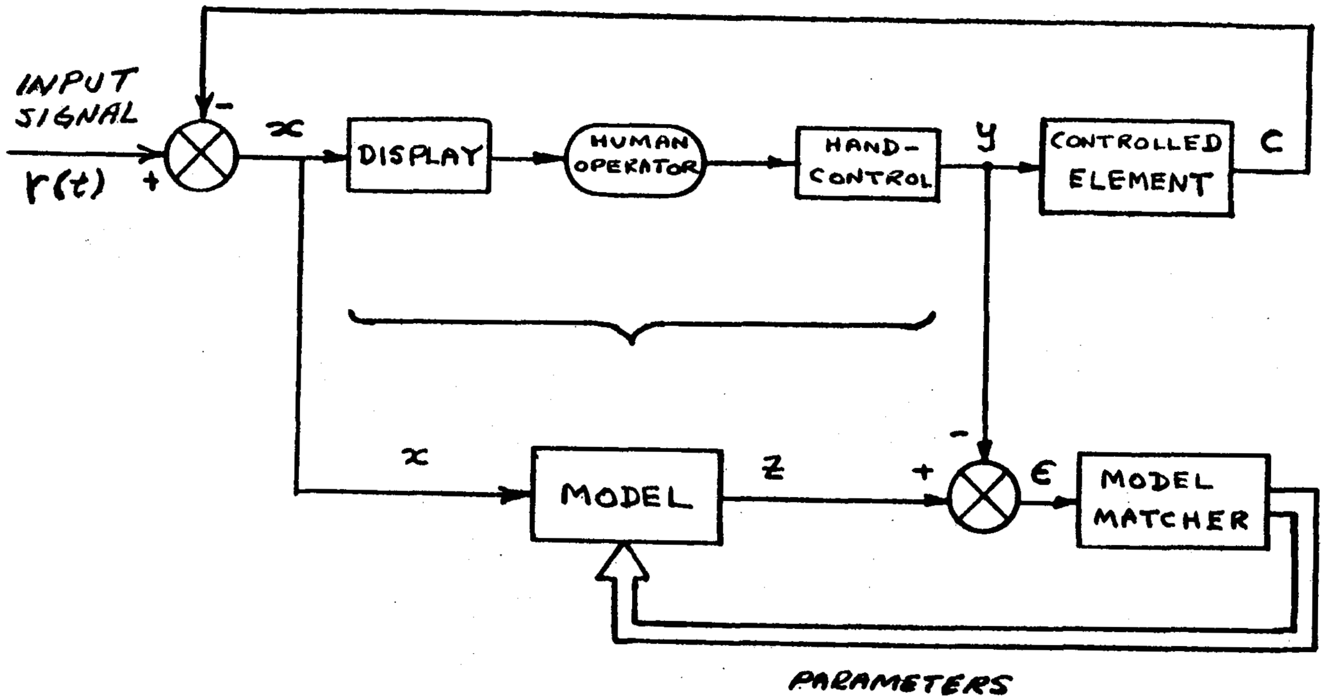

Figure 1. Block Diagram of Single Axis Tracking Task

The semi-iterative technique adjusts sets of two parameters at a time while the other two are held constant. The objective of this approach is to minimize inter-parameter coupling during the adjustment process.

For comparison of the various adjustment processes and the model matching quality obtained a figure of merit was introduced which defines the fraction of the system output y matched by the model output z in terms of signal power. The power match (P) is defined by

$$P = 1 - \frac{\int_0^T \epsilon^2 \, dt}{\int_0^T y^2 \, dt} \tag{1}$$

The time interval used in the integrals of equation (1) can be either the entire time period from the beginning of parameter adjustment or a selected time period after the parameters have attained final values. The latter tends to yield a higher value for P.

3. MODIFIED ERROR CRITERION

The square law error criterion

$$f_1 = (\epsilon + q\dot{\epsilon})^2 \tag{2}$$

previously employed in the parameter optimization process has the disadvantage of a shallow minimum. This causes a relatively large uncertainty in the final parameter values, since the error criterion in practice does not register small deviations of the parameters from the theoretical optimum. An increase of the adjustment gain constant K tends to reduce the uncertainty level but also tends to cause instability of the adjustment process if the error and hence the slope of the error criterion is large.

A modified error criterion having a limited slope for large deviations from the optimum was adopted to overcome this difficulty. This criterion function $f_2(\epsilon, \dot{\epsilon})$ and its slope $\partial f_2/\partial\epsilon$ is shown in Figure 2. It can be expressed mathematically by

$$f_2 = \begin{cases} (\epsilon + q\dot{\epsilon})^2 & \text{if} \quad |\epsilon + q\dot{\epsilon}| < L \\ |\epsilon + q\dot{\epsilon}| - m & \quad\quad |\epsilon + q\dot{\epsilon}| \geq L \end{cases} \tag{3}$$

where m = L (1 - L).

Independent choice of the break point L, the center slope K_c and the rate coefficient q permits adaptation of the error criterion for optimum model matching performance. For a given center slope and break point, the limit of $\partial f_2/\partial\epsilon$ is determined by $M = K_c L$. Typical values used during the experimental study were

$$K_c = 5$$

$$L = 6 \text{ deg}$$

$$q = 0.5 \text{ sec}$$

If the center slope is increased to values of 30 and above an extremely sharp definition of the minimum of f_2 is obtained. Model

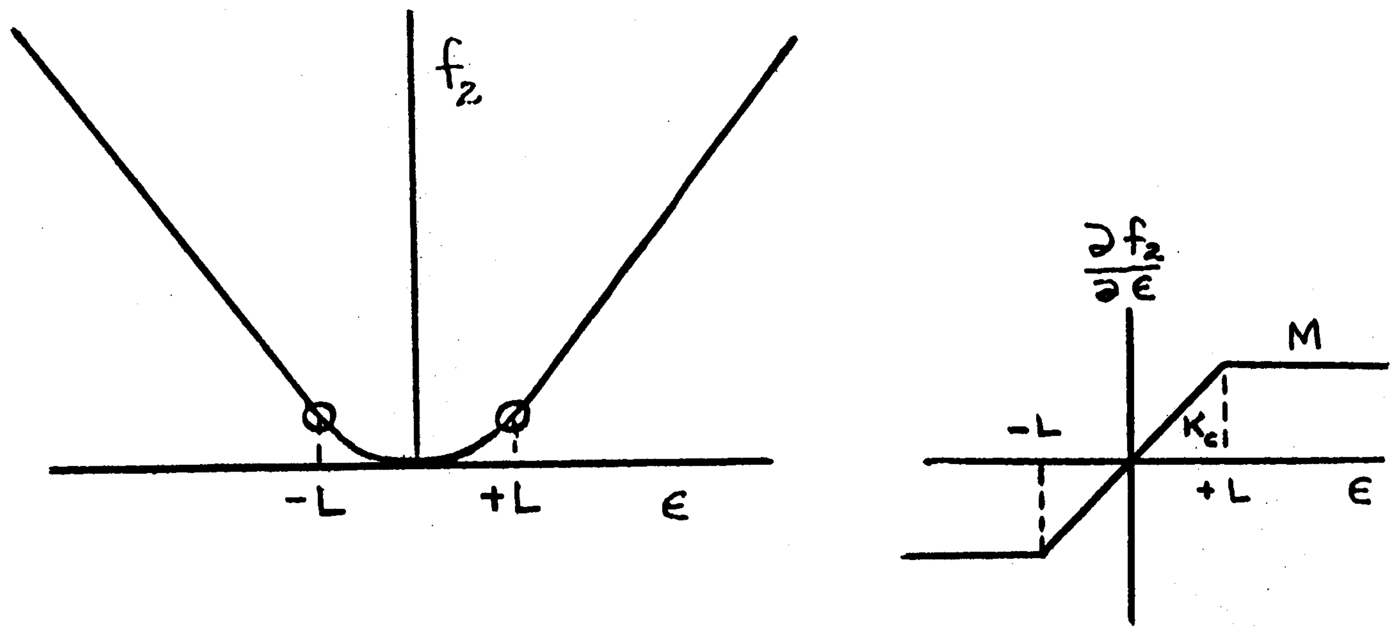

Figure 2. Modified Error Criterion f_2 and Derivative

matching of the human operator to better than 90 percent was possible in some cases (see below). For large center slopes f_2 approximates the absolute error criterion

$$f_3 = |\epsilon + q\dot{\epsilon}| \tag{4}$$

without the attendant problems of switching transients at $\epsilon + q\dot{\epsilon} = 0$ and of limit cycles occurring in the adjustment loops.

The error rate coefficient q used in f_2 can be reduced below the value of 0.5 which had been found optimum for the square law criterion f_1 (see Reference 1) since the effective loop gain for large errors is very much reduced by imposing the limit L on the gradient. The optimum choice of q is a function of K_c and L, but has not been derived in general. It was observed that large values of q tend to produce undesirable sign reversals of the gradient

$$\frac{\partial f_2}{\partial \alpha_1} = 2(u_1 + q\dot{u}_1) \operatorname{sgn} (\epsilon + q\dot{\epsilon}) \tag{5}$$

for large instantaneous $\dot{\epsilon}$ when ϵ and $\dot{\epsilon}$ have opposite sign. This led to a choice of small q-values, e.g., $q = 0.05$ for most of the first order model matching experiments.

4. EXPERIMENTAL STUDY OF SECOND ORDER MODELS

4.1 Modeling of Known Systems

The effectiveness of the modified error criterion function was studied for a second-order known system with parameters

$$a_1 = 12 \text{ sec}^{-1}$$
$$a_2 = 20 \text{ sec}^{-2}$$
$$a_3 = 15 \text{ sec}^{-1}$$
$$a_4 = 10 \text{ sec}^{-2}$$

The model equation was of the form

$$\ddot{z} + \alpha_1\dot{z} + \alpha_2 z = \alpha_3\dot{x} + \alpha_4 x \tag{6}$$

Satisfactory values for the model matcher constants

$$K_c = 5$$
$$q = 0.5 \text{ sec}$$
$$L = 6 \text{ deg}$$

were obtained by observing the power match and the stability of parameter adjustment. These values were subsequently used in the modeling of actual human tracking data.

4.2 Models of Human Operators

Model matching on human tracking data, using the model equation (6) encountered a number of difficulties. At the end of a tracking run of two minutes the parameters had not attained steady state. A sequence of model matching runs on the same tracking data was performed, each run using the end values of the previous run as initial values. In some cases the parameters continued to drift throughout the iterative sequence as illustrated in Figure 3. The use of semi-iterative parameter adjustment produced similar results as shown in Figure 4.

A start from various sets of initial parameters resulted in inconsistent steady state parameter values although approximately the same power match of 80 percent was obtained with these different sets of parameters.

This case of poorly defined parameters was further investigated experimentally. It was found that parameter ratios such as α_1/α_2 and α_3/α_4 maintained relatively consistent values in repeated model matching runs in spite of the indeterminacy of the individual α_i (see also Section 6).

5. EXPERIMENTAL STUDY OF FIRST ORDER MODELS

The revised model equation

$$\dot{z} + \beta_1 z = \beta_2 \dot{x} + \beta_3 x \tag{7}$$

was used in an attempt to simplify the modeling procedure in cases where the second order presentation (6) gave inconsistent or unsatisfactory results due to small second order terms*. The constants of the criterion function used in this study were

$$K_c = 60$$

$$q = 0.05 \text{ sec}$$

$$L = 9 \text{ deg}$$

The very small value of q required for this part of the study has been previously explained (see Section 3). A systematic study of parameter convergence and accuracy was performed in terms of a first order system with known coefficients and is reported in Reference 5.

Model matching of actual human tracking data using a first order model of the form (7) resulted in stationary and repeatable parameter values. A power match of approximately 80 percent was obtained in the case of a first order model, nearly the same as for a second order model.

---

* The problem of small parameters is analyzed in Reference 3.

Since the stability characteristics of a first order model matcher are much less critical it was possible to obtain even better power matches, in excess of 90 percent, when the adjustment gain K_c was raised to values of 60 or 90. The results are shown in Figures 7, 8, and 9.

In the case of extremely high adjustment gains and nearly zero modeling error it was observed that the resulting model parameters oscillate rapidly about average final values. The average values are the same as those obtained for lower adjustment gain K_c where a power match of 75 to 85 percent was measured (see specimen shown in Figure 8). Three possible explanations have been advanced for the observed parameter fluctuations:

1) The variations in β_i are primarily caused by the inhomogeneous terms

$$K_c \epsilon(0)\, u_i \quad \text{or} \quad K_c\, [\epsilon(0) + q\dot{\epsilon}(0)]\, [u_i + q\dot{u}_i] \tag{8}$$

in the adjustment equation (see Reference 1, page 72). These terms are due to continued excitation of the model matcher in the presence of non-zero residual error $\epsilon(0)$ accentuated by large adjustment gain K_c*.

2) The mismatch in model format due to the attempt of modeling the human operator by a first order equation (7) enforces continuous time variations of the resulting parameters under the constraint of nearly zero model matching error.

3) The parameter variations reflect actual changes in human operator parameters.

Further research will be required to substantiate these hypotheses. A combination of several of these effects may be the cause of the observed parameter fluctuations. It was concluded that cause (3) is least likely because actual fluctuations of pilot dynamics would not be traced as rapidly by the model matcher (having time constants on the order of one or more seconds) as the fluctuations portrayed in Figure 7.

* This effect has been referred to as "scalloping" in previous discussions.

6. PARAMETER INDETERMINACY

A difficulty in parameter identification can arise if the system parameters assume certain critical ratios which cause indeterminacy. As an illustration consider the system equation

$$\ddot{y} + a_1\dot{y} + a_2 y = a_3\dot{x} + a_4 x \tag{9}$$

having the transfer function

$$\frac{Y(s)}{X(s)} = \frac{a_3 s + a_4}{s^2 + a_1 s + a_2} \tag{10}$$

For input signals of low frequency equation (10) is approximated by

$$\frac{Y(s)}{X(s)} \cong \frac{a_3 s + a_4}{a_1 s + a_2} \tag{11}$$

If the known parameters have values related by

$$\frac{a_3}{a_1} = \frac{a_4}{a_2} = C_1 \tag{12}$$

equation (10) simply becomes

$$\frac{Y(s)}{X(s)} = C_1$$

where C_1 is the zero-frequency gain of (10).

The corresponding model equation is transformed similarly into

$$\frac{Z}{X}(s) \cong \frac{(\alpha_3 s + \alpha_4)}{(\alpha_1 s + \alpha_2)} \tag{13}$$

A set of system parameters which are related in accordance with equation (12) cannot be uniquely identified by model matching because the requirement

$$Z(s) = Y(s)$$

can be satisfied in good approximation by any sets of parameters α_i related by

$$\frac{\alpha_3}{\alpha_1} = \frac{\alpha_4}{\alpha_2} = C_1 \tag{14}$$

i.e., the α-parameters will not necessarily be equal to the known a-parameters. For high excitation frequencies the approximations (11) and (13) are not valid and hence the indeterminacy of parameters α_i will disappear.

Figure 5 illustrates a plot of model parameters in the α_1, α_3 plane and in the α_2, α_4 plane. The lines $\alpha_3 = C_1\alpha_1$ and $\alpha_4 = C_1\alpha_2$ are loci of indeterminate parameter pairs. The α_i actually obtained by the computer depend largely on the choice of initial values $\alpha_i(0)$. In practice, even system parameters located in the <u>vicinity</u> of these loci can cause indeterminacy problems on the computer. In the presence of computer noise, a continuous drift of the parameters along the loci, or in their vicinity, is to be anticipated.

This theory explains several experimental examples obtained on the computer where the parameters a_i of a "synthetic pilot" were chosen inadvertently in equal ratios $a_3/a_1 = a_4/a_2 = 2$. Good power matches were obtained in the parameter identification process although the α_i obtained were not unique and depended on initial values $\alpha_i(0)$.

Figure 6 illustrates a case of poor determinacy of parameters obtained from human tracking data. Repeated model matching resulted in continuous monotonic drift of the individual parameters. The final ratios α_1/α_3 and α_2/α_4 obtained were approximately equal.

A similar problem of parameter indeterminacy can also arise in a first order model matcher. If the system and model equations are given by

$$\dot{y} + b_1 y = b_2\dot{x} + b_3 x$$

$$\dot{z} + \beta_1 z = \beta_2\dot{x} + \beta_3 x$$

having the transfer functions

$$\frac{Y}{X}(s) = \frac{(b_2 s + b_3)}{(s + b_1)} \tag{15}$$

and

$$\frac{Z}{X}(s) = \frac{(\beta_2 s + \beta_3)}{(s + \beta_1)} \tag{16}$$

respectively, parameter indeterminacy will occur in model matching if the system parameters are related by

$$b_2 = \frac{b_3}{b_1} = C_2 \tag{17}$$

In this singular case the transfer function is frequency-independent having the gain C_2 at all frequencies.

Note that in this case the indeterminacy arises solely due to a singular combination of parameters, whereas in the second order case the indeterminacy is caused by singular parameters in combination with low excitation frequency. Since a human operator's transfer function model cannot be truly frequency-independent, the singular condition expressed by (17) will not actually prevail at high excitation frequencies.

Parameter indeterminacies induced by singular conditions of the form (12) or (17) can possibly be circumvented if the system output z is modified by insertion of a low pass filter such that

$$\frac{Z'}{X} = \frac{Z}{X} \cdot G_F(s) \cong \frac{a_3 s + a_4}{a_1 s + a_2} \quad \frac{1}{\tau s + 1} \tag{18}$$

The resulting transfer function

$$\frac{a_3 s + a_4}{a_1 \tau s^2 + (a_1 + a_2 \tau) s + a_2} \tag{19}$$

is no longer singular, having coefficient ratios

$$\frac{a_3}{a_1 + a_2 \tau}, \quad \frac{a_4}{a_2}$$

which are inequal under the condition (12), for $\tau \neq 0$. After determining the modified parameters of (18) the effect of the filter can be deducted to obtain the original coefficients. Further investigation of this method will be conducted if necessary in the forthcoming model matching program.

7. CONCLUSIONS

The above experimental studies have provided significant new insight into capabilities and limitations of the continuous parameter adjustment technique. The findings necessitated further analysis of the parameter determinacy problem which can occur in human operator models and which can be recognized by the behavior of certain parameter ratios.

A separate analysis of small parameter problems, of relative parameter sensitivity, and of the stability and convergence characteristics of the model matcher were the outgrowth of this study (References 3, 4, 5). The first order model parameters resulting from the tracking data investigated here are presented in Reference 2 in greater detail.

The new optimization criterion developed during this study constistutes a major improvement and will be used under Tasks 2 and 3 of human data evaluation.

DISTRIBUTION

G. A. Bekey
W. A. Finley
R. E. Rose
L. G. Summers
E. P. Todosiev

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5. "Experimental Design for Tasks 1, 2, and 4 of NASA Contract NAS 1-4419," No. 9352.2-7, by E. P. Todosiev, dated 12 February 1965.

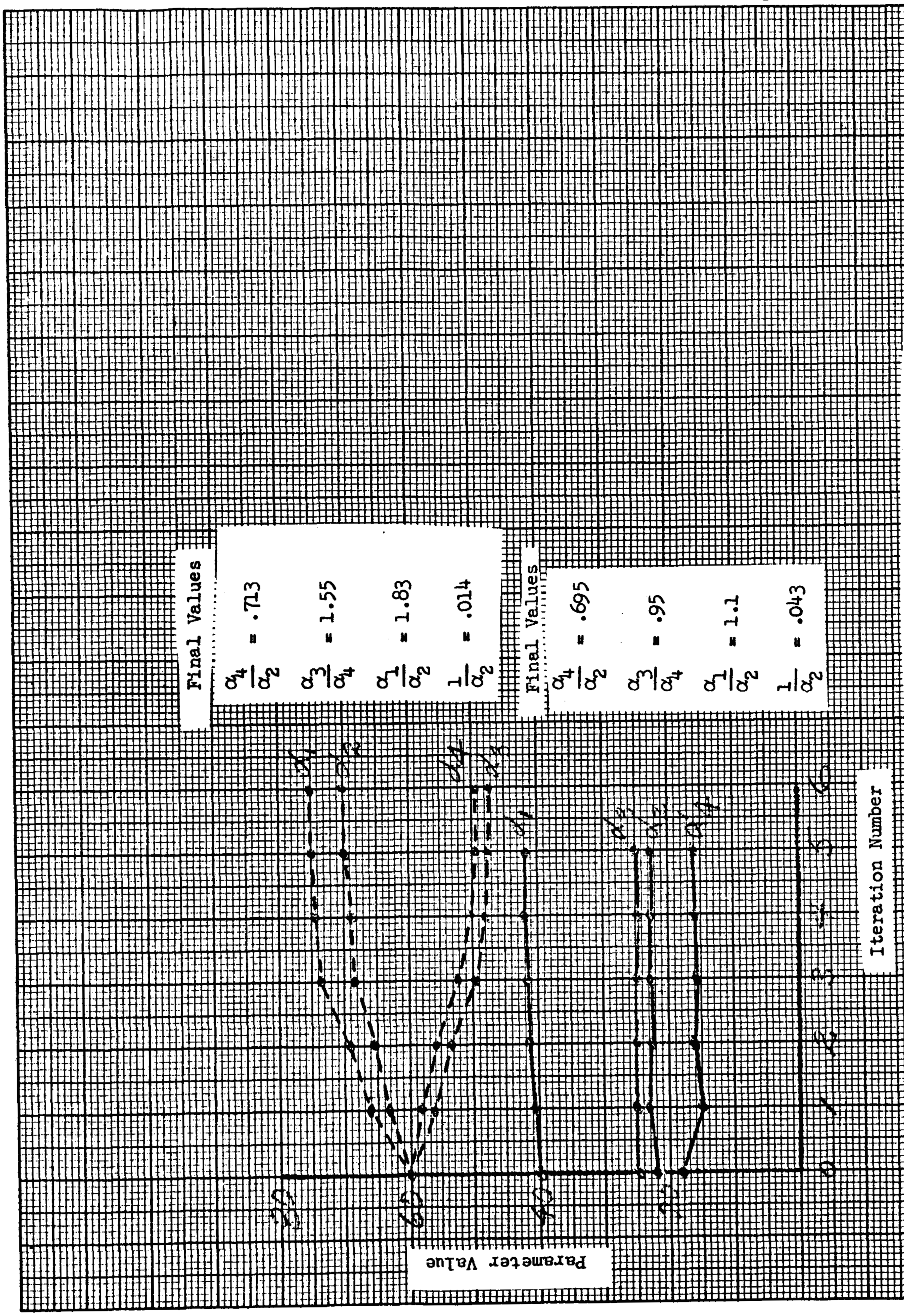

Figure 3. Results of Iterative Parameter Optimization Versus Iteration Cycles

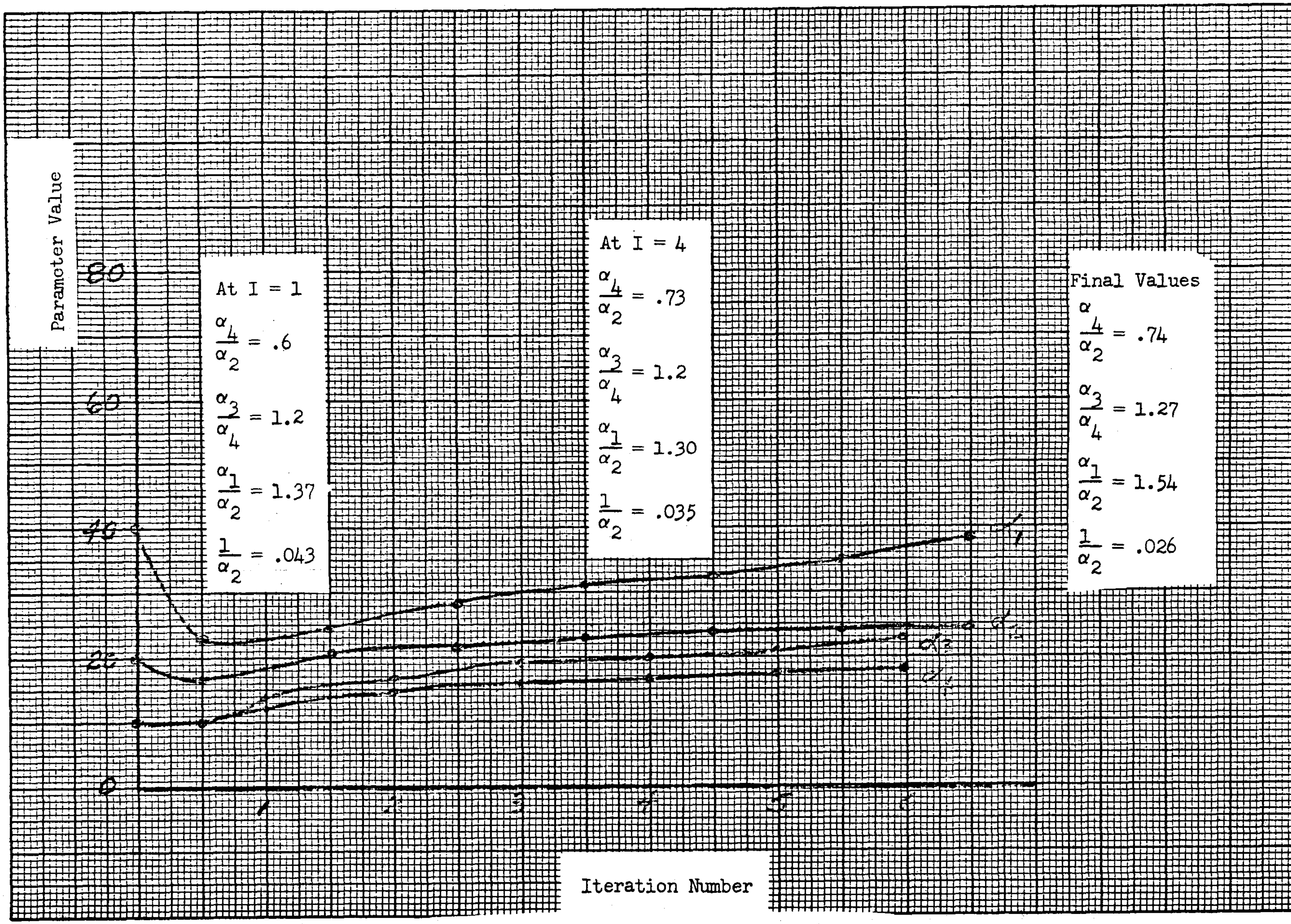

Figure 4. Results of Semi-Iterative Parameter Optimization Versus Iteration Cycles

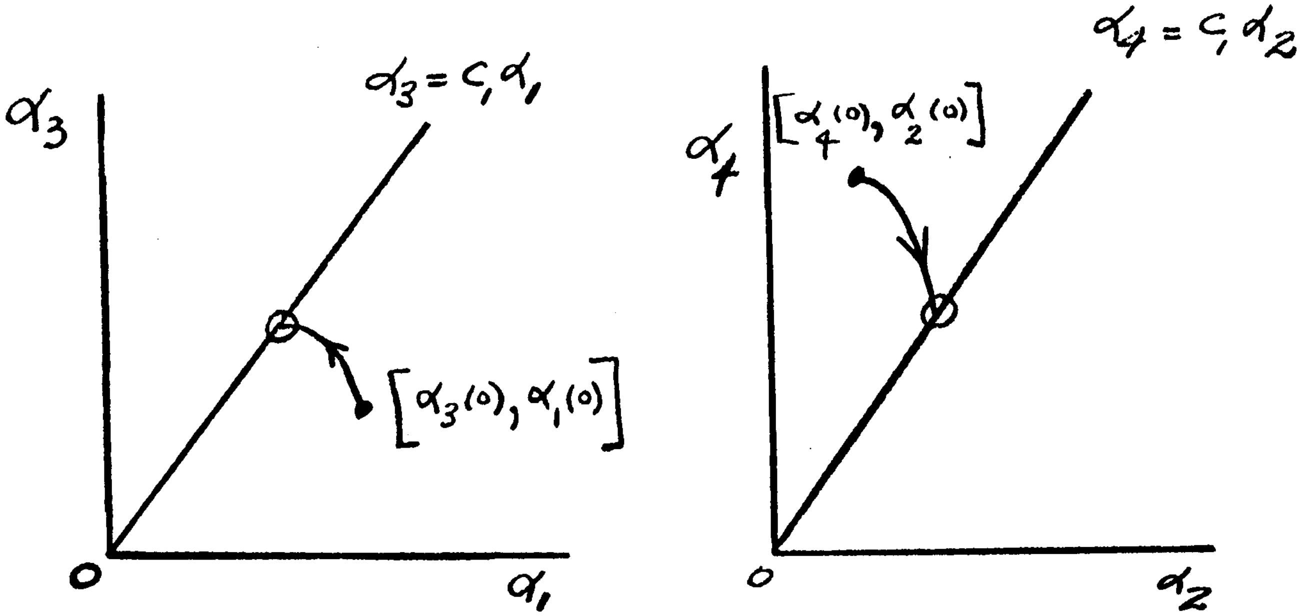

Figure 5. Loci of Indeterminate Parameter Pairs in α_1, α_3 and α_2, α_4 Planes

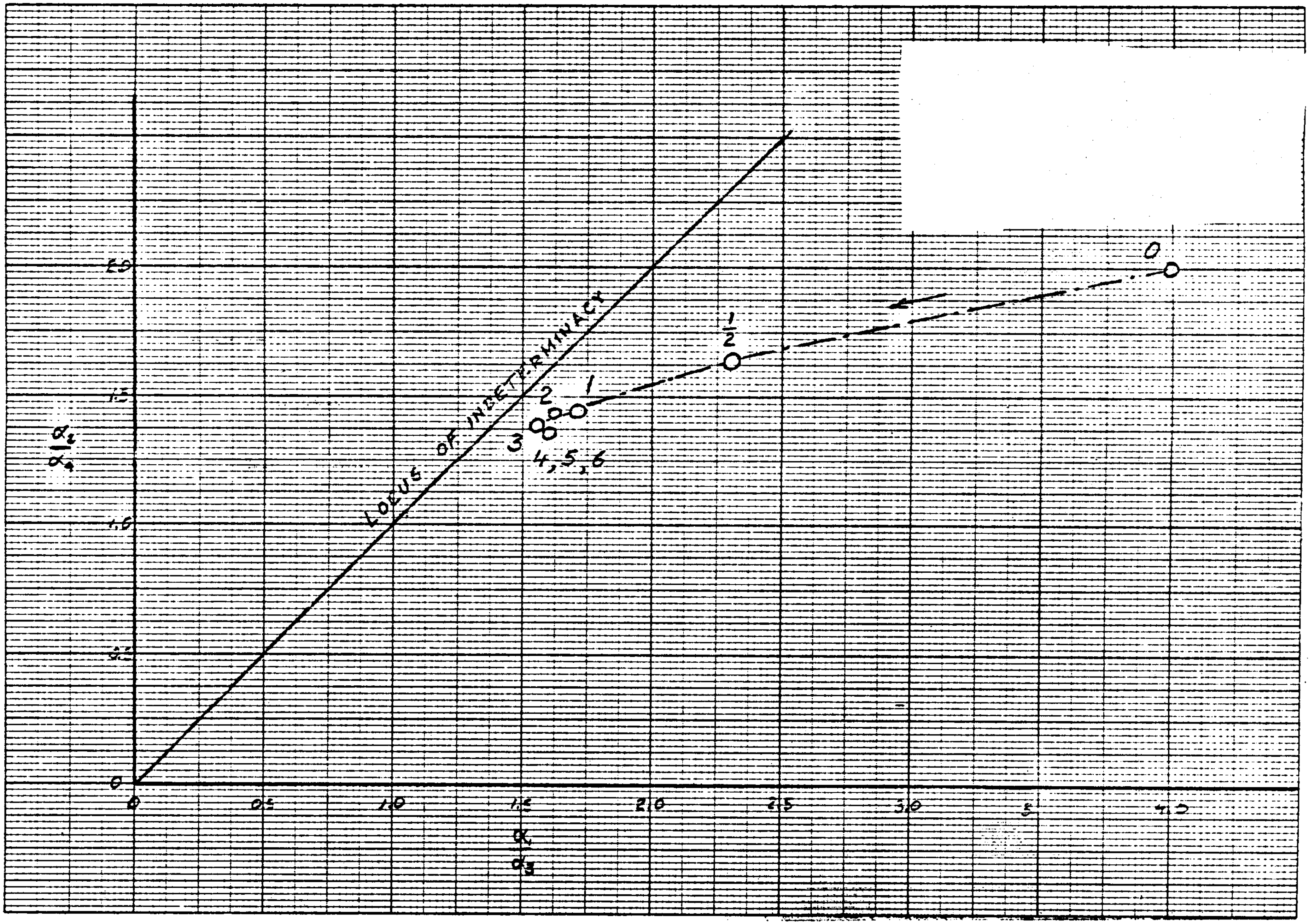

Figure 6. Parameter Ratios Obtained by Semi-Iterative Process (cf. Figure 4)

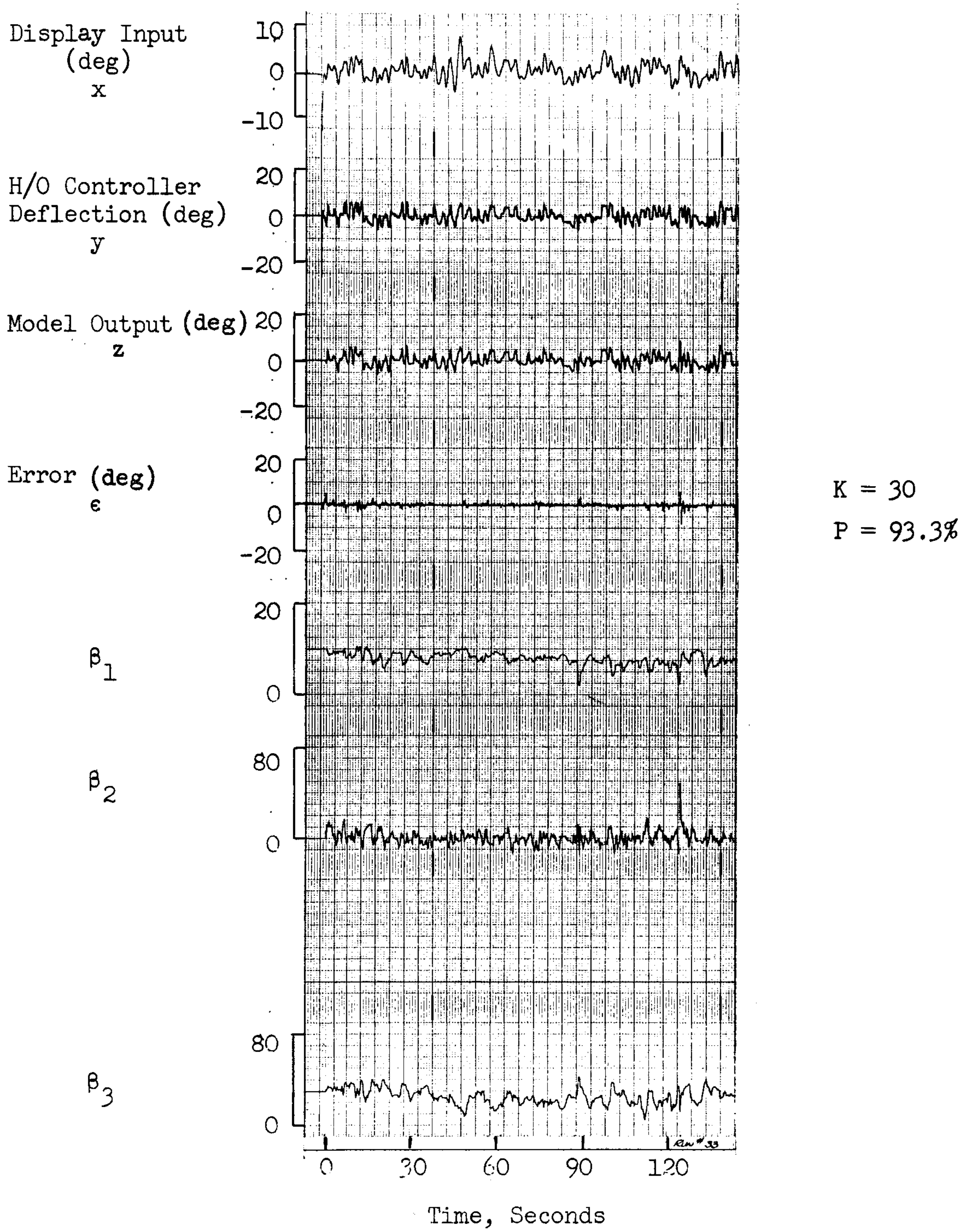

Figure 7. First Order Model Matching (High K_c)

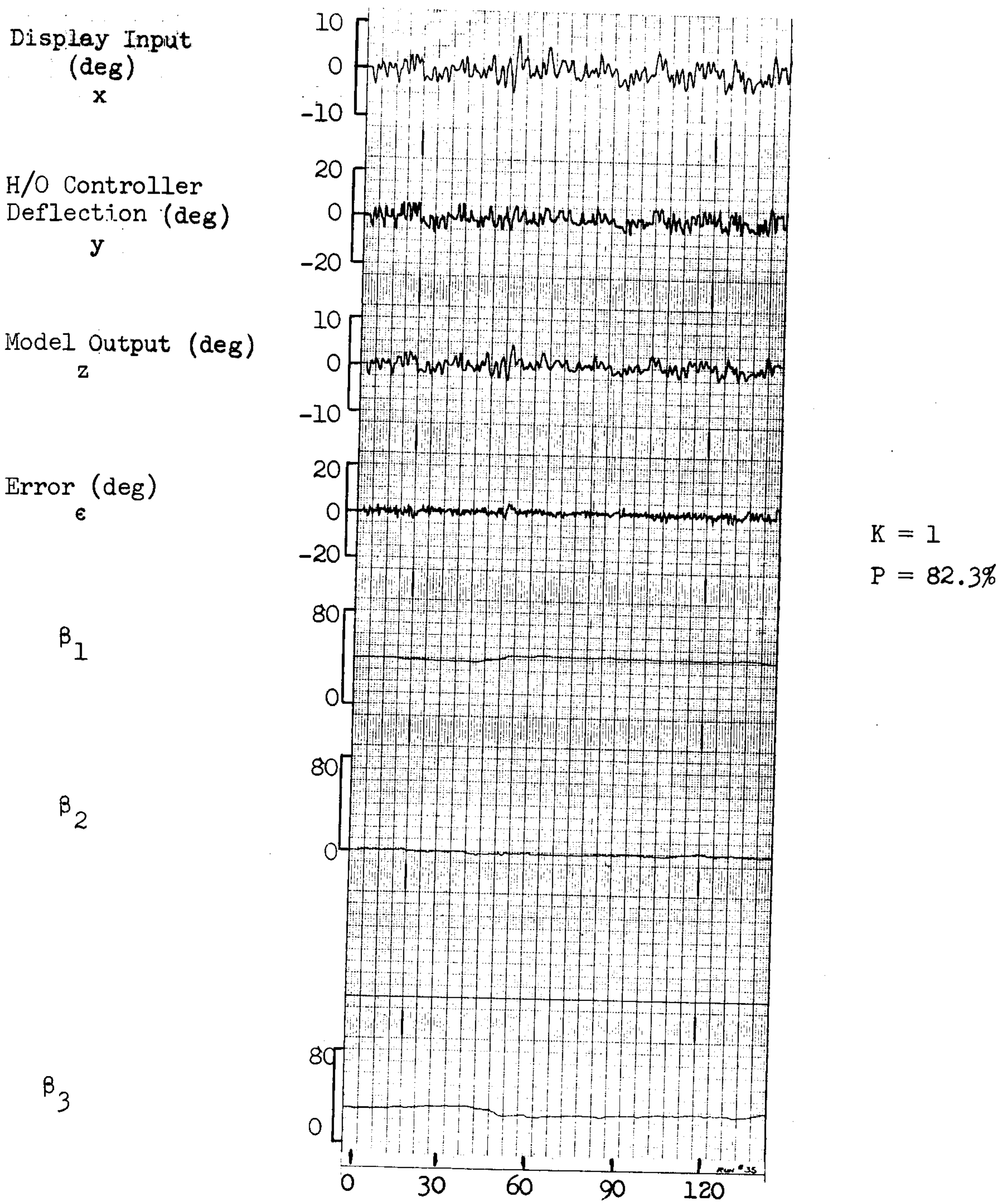

Figure 8. First Order Model Model Matching (Low K_c)

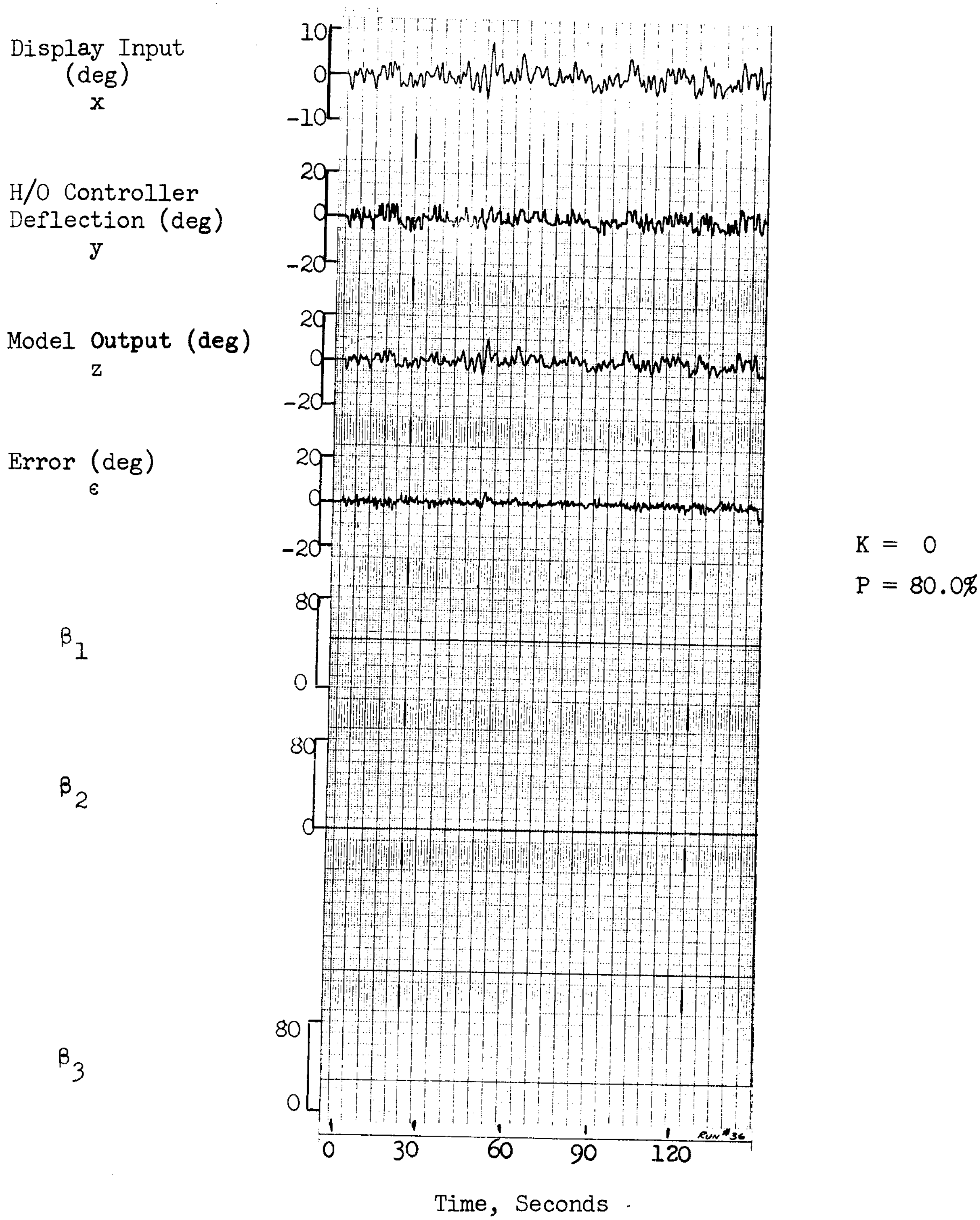

Figure 9. First Order Model Matching (Fixed Parameters)

TO: Distribution CC: DATE: 25 March 1965

SUBJECT: Convergence Study of First Order Model Parameters FROM: E. P. Todosiev

BLDG. R2 ROOM 1186 EXT. 12 250

A first order differential equation with known constant coefficients was formulated to be experimentally analyzed using a first order model-matcher. The purpose of the analysis was to determine the effect of adjustment gain and parameter initial conditions on the convergence characteristics of the model parameters. Parameter accuracy and repeatability was also to be a major consideration.

A conventional first order model-matcher with a modified criterion function was used. Details of the criterion function may be found in Reference 1. In this study the criterion constants were

$$K = 60$$
$$q = 0.05 \text{ sec.}$$
$$L = 9 \text{ degrees}$$

where q is the weighting function of $\dot{\epsilon}$ and L is the limiter value. Figure 1 illustrates the criterion function used. If the first order differential equation with constant coefficients is considered as the describing equation for a synthetic pilot in a tracking task, then the synthetic pilot may be regarded as a black box with an input and output. If the pilot and model-matcher inputs are denoted by x and their respective outputs by y and z, then the differential equations describing the dependence of y and z on x are given by

$$\dot{y} + b_1 y = b_2 \dot{x} + b_3 x \qquad (1)$$

$$\dot{z} + \beta_1 z = \beta_2 \dot{x} + \beta_3 x \qquad (2)$$

where b_1, b_2 and b_3 are the (constant) coefficients of the pilot and β_1, β_2, and β_3 are the model parameters. The model parameters are adjusted by the model-matcher so as to make $\epsilon = z - y$ equal to zero at which time the β parameters become equal to the b parameters and the identification of the pilot parameters is complete. For this study the pilot parameters were chosen to have values representative of comparable human transfer functions. Specifically these values were

$$b_1 = 40 \text{ sec}^{-1}$$

$$b_2 = 15$$

$$b_3 = 25 \text{ sec}^{-1}$$

Equation (1) may be written in the transfer function form

$$\frac{Y}{X}(s) = \frac{K_o (T_1 s+1)}{(T_2 s+1)}$$

where

$$K_o = \frac{b_3}{b_1} = 0.625$$

$$T_1 = \frac{b_2}{b_3} = 0.6 \text{ sec}$$

$$T_2 = \frac{1}{b_1} = 0.025 \text{ sec}$$

and s is the Laplace operator. A two minute tape recording of a tracking error history obtained from a compensatory tracking experiment was used as the input x for all phases of the study. The convergence study was initiated by first investigating the repeatability characteristics of the model-matcher for various values of the initial conditions of the β parameters.

EFFECT OF INITIAL PARAMETER VALUES

A random choice for the initial parameter values will yield a criterion function whose magnitude will also be of a random value. To circumvent this dilemma, the initial conditions were chosen such that the criteria function would have a large magnitude by assigning zero initial conditions to β_2 and β_3. The parameter β_1 must be non-zero to keep the model transfer function gain from approaching infinity. Specifically β_1 was initially chosen to have values which were either high or low by 50% with respect to the known value for b_1. With the above described initial conditions, a repeatability experiment was performed on the model-matcher to determine the effect of these initial conditions on the repeatability characteristics. In these experiments, the model-matcher was allowed to operate on the input data for short lengths of time. Model-matcher gains of 30, 60 and 90 were used. Figure 2 shows the poor repeatability characteristics for the β parameters when $\beta_1(0) = 0.5\ b_1$ and the adjustment gain was 60. With $\beta_1\ (0) = 1.5\ b_1$, the parameter repeatability was markedly better as shown in Figure 3. Adjustment gains of 30 and 90 yielded similar results. It was concluded from the experimental results that β_1 should be above its time value initially if good repeatability was to be expected. No theoretical reasons are offered for this behavior at this time. These initial conditions were used in all of the subsequent experimental measurements.

LONG TERM CONVERGENCE

In operation the model-matcher will cause the β parameters to converge on their true values if sufficient time is available. A typical time history of this process is shown for one parameter in Figure 4. Note that the parameter converges approximately to the true value in two distinct steps. Initially the convergence is very rapid and consequently this portion of the convergence has been termed short term convergence. After this rapid initial convergence, the parameter requires a long settling time before it reaches a steady-state value (i.e. long term convergence). The initial convergence is rapid because the error ϵ is large and consequently the slope of the criterion function is large. However, when the error becomes small (point A on Figure 4), the resultant criterion function has a small slope with respect to ϵ which decreases the convergence rate.

An experimental analysis was conducted on the long-term parameter convergence to determine the effect of adjustment gain and matching time on the parameter accuracy. Figure 5 shows the percentage error in the β parameters for the various adjustment gains where the parameter values were determined upon completion of a 2-minute run. Percentage errors for the equivalent transfer function parameters are also shown in Figure 5. Clearly, Figure 5 indicates that the β parameters may be obtained with a percentage accuracy of $\pm 6\%$ or better while the transfer function parameters may be determined to an accuracy of $\pm 4\%$. In particular the parameter K_o may be determined to an accuracy of better than 0.5%.

In an attempt to increase the accuracy of the convergence process, the same data was run through the model-matcher a number of times. Four adjustment gains of 10, 30, 60 and 90 were used and in the final parameter values of 1 run were made the initial conditions for the subsequent run. Figure 6 indicates the dependence of β parameter percentage error on the number of replications R as well as the gain used. In general, the percentage error was greater after two replications. In cases where three replications were made, the percentage error had either reached a plateau (for $K = 60$) or was approaching one (for $K = 10$). All parameters had approximately the same percentage error and were predominantly negative. Percentage errors were also calculated for the equivalent transfer function parameters and are shown in Figure 7. Again, the use of replications is apparently not warranted as the accuracy is not increased substantially. The one exception occurs when the gain is 60. Here a definite increase in accuracy for the κ_o and τ_1 parameters was obtained if replications were made. Comparison of the accuracies for the β and transfer function parameters indicates that the transfer function parameters are again more accurately determined (especially for κ_o and τ_1). This result is due to the fact that the transfer function parameters are ratios of β parameters. Since the β parameters have errors which are consistently negative and approximately equal, it follows that their ratios will be much more accurate with the sole exception of parameter τ_2 which is not a ratio but a reciprocal. Figure 7 clearly shows that τ_2 is much less accurate than κ_o or τ_1.

SHORT TERM CONVERGENCE

In the conduction of the long term convergence experiments it was noted that the error was very close to zero at the end of the short term convergence period. To determine the parameter accuracy at this point, an experiment was conducted in which the short term parameters were found for five randomly chosen points of the same data run previously used. These parameters were then averaged and the RMS value of the percentage error determined. In general, the accuracies were not as good as in the long term case. However, the transfer function parameters with the exception of τ_2 were found to be accurate to 5% over all of the adjustment gains used. Figure 8 compares the accuracy of the β and transfer function parameters. Again, the transfer function parameters are more accurate with the exception of τ_2. This may be explained by the same argument used for the long term convergence study. It is important to realize that the short term parameters are accurate to 10% RMS for K = 90 as their values may be determined in a second or two while the long term parameters require about 60 seconds.

CONTROL REVERSAL

In many manual control tasks it is a common occurrence to find that the human operator will sometimes reverse the sense of his polarity output for a short time even though his signal input has not changed polarity during that time interval. This 180° phase shift is known as a control reversal and at present no method other than direct visual comparison of the input and output signals exists for the detection of control reversals (Reference 2). Since the model-matcher can rapidly converge in the short term sense with a fair accuracy (10%), an experiment was formulated to determine if the model-matcher could monitor the parameter changes that result when a control reversal occurs.

The control reversal was simulated by providing an operational amplifier that could operate serially on the input signal to the synthetic pilot and change its phase by 180°. This scheme could have been used instead to change the phase of the pilot output signal by 180° and thus effect a direction simulation. However, it was found more convenient to use the first method. Figure 9 illustrates the switching system used to simulate the control reversal. Examination of the computer implementation of the pilot indicates that the control reversal will cause the model parameters β_2 and β_3 to change sign from positive to negative and consequently the model transfer function will have a corresponding negative gain K_0.

Using the same input data x, experiments were performed with the model-matcher to determine if it could monitor control reversals. In general, the performance was poor in that the model-matcher parameters would not always converge when the control reversal was simulated. However, many instances did occur when the model-matcher was able to properly detect the control reversal. Figure 10 illustrates one case where a control reversal was simulated at time t_1. At time t_2 the original control polarity was reestablished. The detection of the control reversal was generally rapid (1 or 2 seconds) and in all cases a transient was also introduced into the β_1 parameter during the reversal as shown in Figure 10. Parameters β_2 and β_3 both reversed polarity during the reversal (at time t_1) and at time t_2 both parameters reverted back to their original value. Consequently, the model-matcher was able to follow the control reversal as well as the reversion back to the original control polarity. To determine the accuracy of the model-parameters just after the occurrence of the reversal, another run was made in which the parameters were measured while the model-matcher was in the computer hold mode. Accuracies of these measurements are shown in Table I where the times t_1, t_2 and t_3 are as indicated in Figure 11.

TABLE 1

	K_o	% E	τ_1 (secs)	% E	τ_2 (secs)	% E
t_1	+0.639	+ 2.24	0.570	-5.00	0.0227	-9.20
t_2	-0.632	+ 1.12	0.577	-3.83	0.0231	-7.60
t_3	+0.625	0	0.587	-2.17	0.0191	-23.6

This preliminary study on the detection of control reversals has shown that the model-matcher is capable of detecting control reversals with a fair degree of accuracy. Since control reversals in the real world are of short duration (in the order of seconds), it will be difficult to increase the parameter accuracy as the model-matcher short term accuracy is relatively poor compared to its long term accuracy. However, the most serious limitation of the model-matcher as a control reversal detector is its inconsistent operation. More research will be required to establish if the model-matcher can be used with the confidence in the detection of control reversals.

CONCLUSIONS

An experimental analysis on the convergence characteristics of a first order model-matcher led to the following conclusions:

(1) Parameter adjustment repeatability was good when $\beta_1\ (o) = 1.5\ b_1$ and $\beta_2(o) = \beta_3\ (o) = 0$.

(2) For long term convergence, the β parameters may be obtained with a percentage accuracy of $\pm 6\%$ while the transfer function parameters may be determined to an accuracy of $\pm 4\%$.

(3) Use of replications does not substantially decrease the long term convergence error.

(4) No optimum gain was found for long term convergence.

(5) For short term convergence, both the β and transfer function parameters may be determined with an accuracy of 10% (RMS) at an adjustment gain of 90.

(6) The optimum adjustment gain for short term convergence was 90 (the highest value used).

(7) For both long and short term convergence, the transfer function parameters may be obtained with a better percentage accuracy than the β parameters except for the case of τ_2 for which no significant difference occurs.

(8) The transfer function parameter K_o may be determined with the greatest precision (0.5% for long term convergence and 2% for short term).

(9) The model-matcher can detect control reversals but the detection is sensitive to the instantaneous value of the input x and is not consistent.

Direct application of these results to the prediction of model-matcher performance on differential equations with unknown coefficients and of an order other than one, cannot be justified from the experimental analysis as the analysis was only concerned with an equation of order one with known constant coefficients. If the unknown coefficients are slowly time-variant it may be possible for the model-matcher to follow the variation in the unknown parameters with a fair degree of accuracy as the model-matcher does exhibit a good short term parameter convergence accuracy.

An analytical study of the sensitivities of the β and transfer function parameters has been made to explain the difference in behavior of the two sets of parameters. This analysis in general supports the experimental work reported here and may be found in Reference 3.

E. P. Todosiev

EPT:js

Distribution

G. A. Bekey
W. A. Finley
R. E. Rose
L. G. Summers
E. P. Todosiev

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(1) 9350.6-157 "Preliminary Modeling Experiments on Human Tracking Data", H. F. Meissinger, R. E. Rose, E. P. Todosiev, March 29, 1965.

(2) "Adaptive Dynamic Response Characteristics of the Human Operator in Single Manual Control", L. R. Young et al., IEEE Transactions on Human Factors in Electronics, Vol. HFE-5, September 1964.

(3) 9350.6-153 "Relative Sensitivity of Human Pilot Model Parameters", H. F. Meissinger, February 26, 1965.

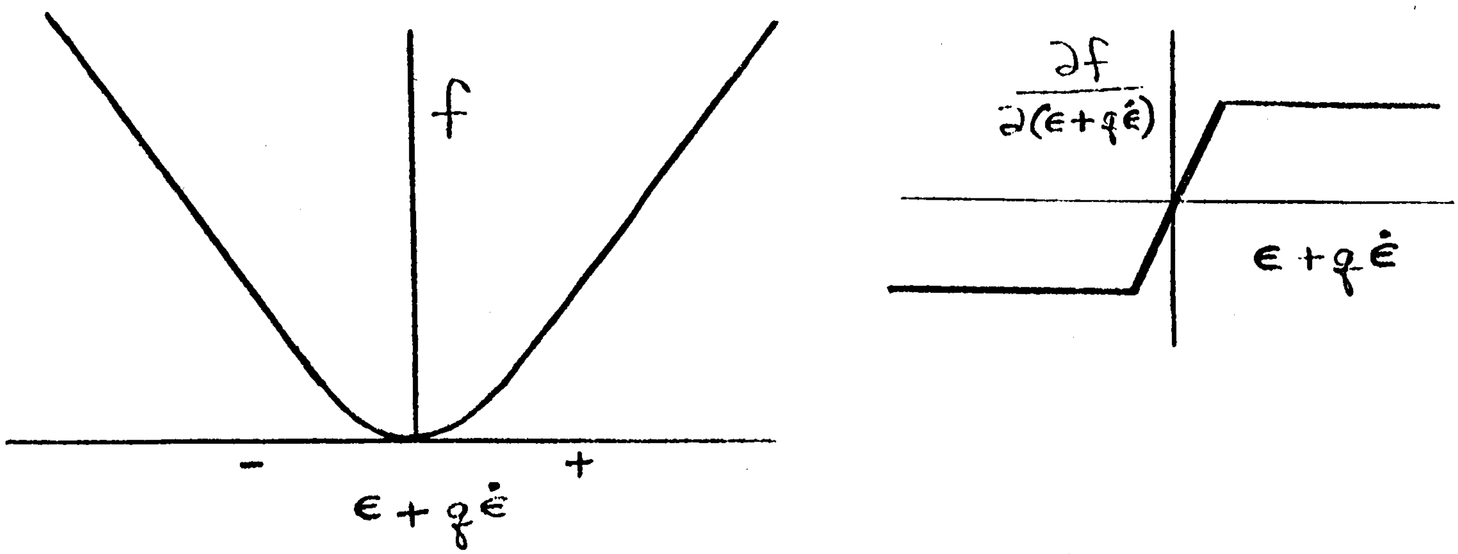

Figure 1. Criterion function

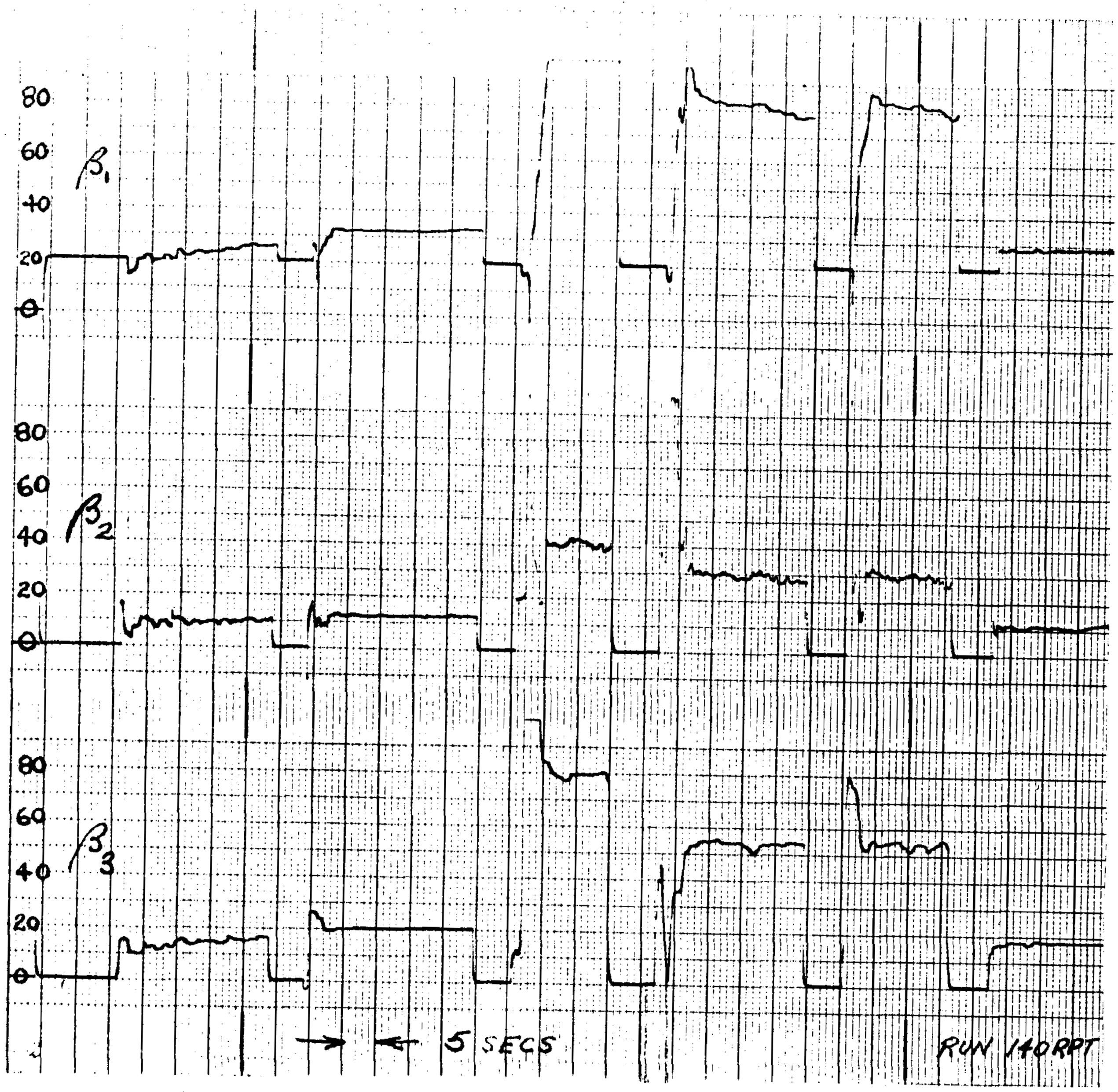

Figure 2. Parameter repeatability ($\beta_1(o) = 20$)

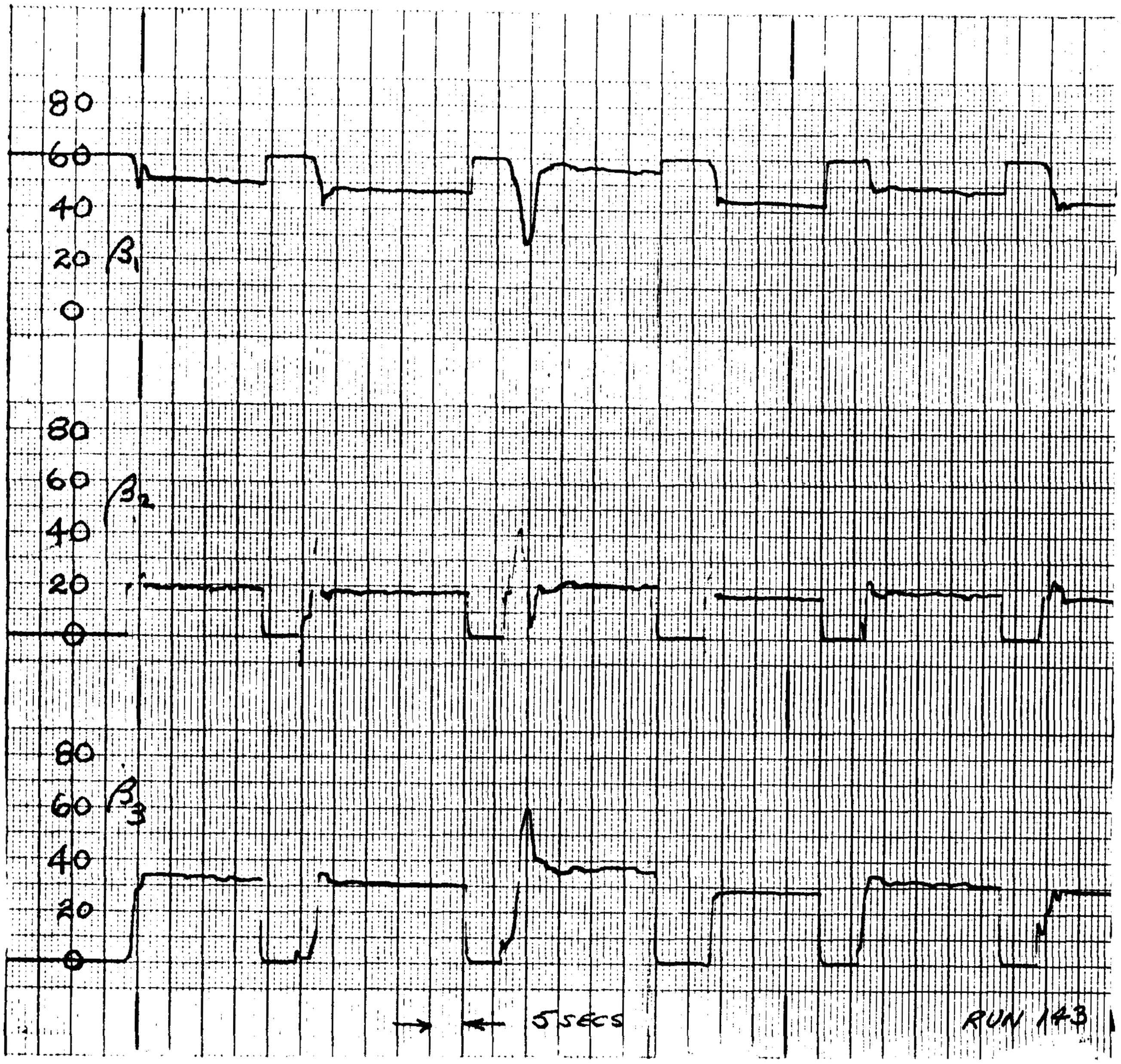

Figure 3. Parameter repeatability ($\beta_1(o) = 60$)

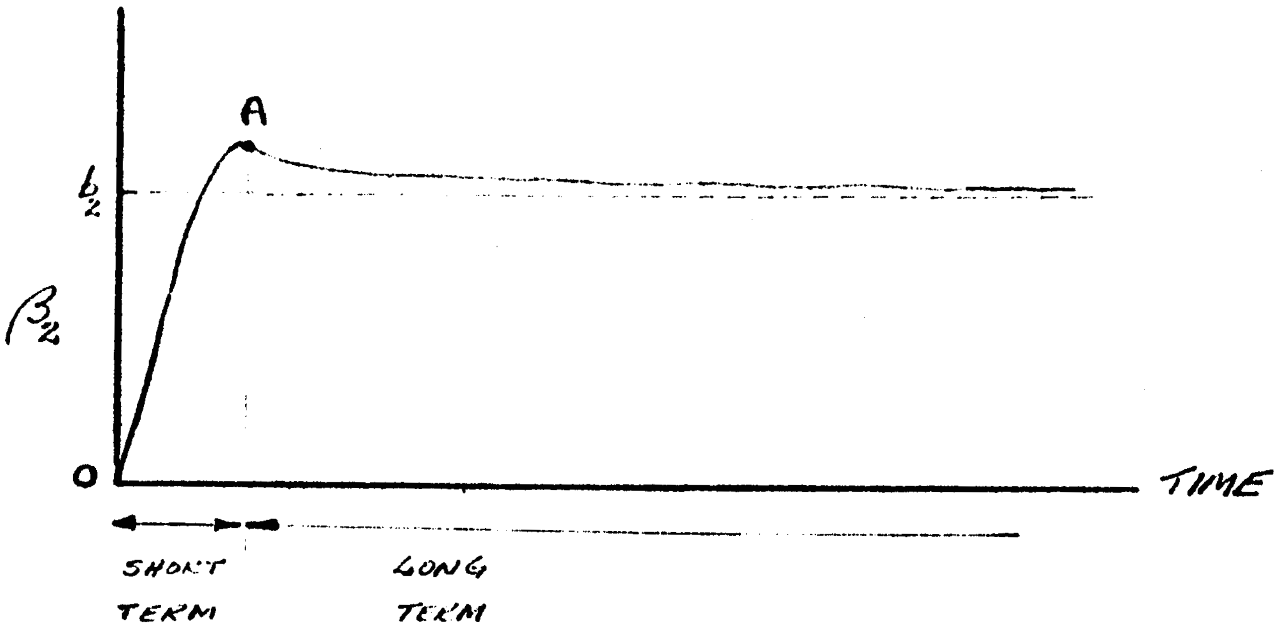

Figure 4. Parameter convergence

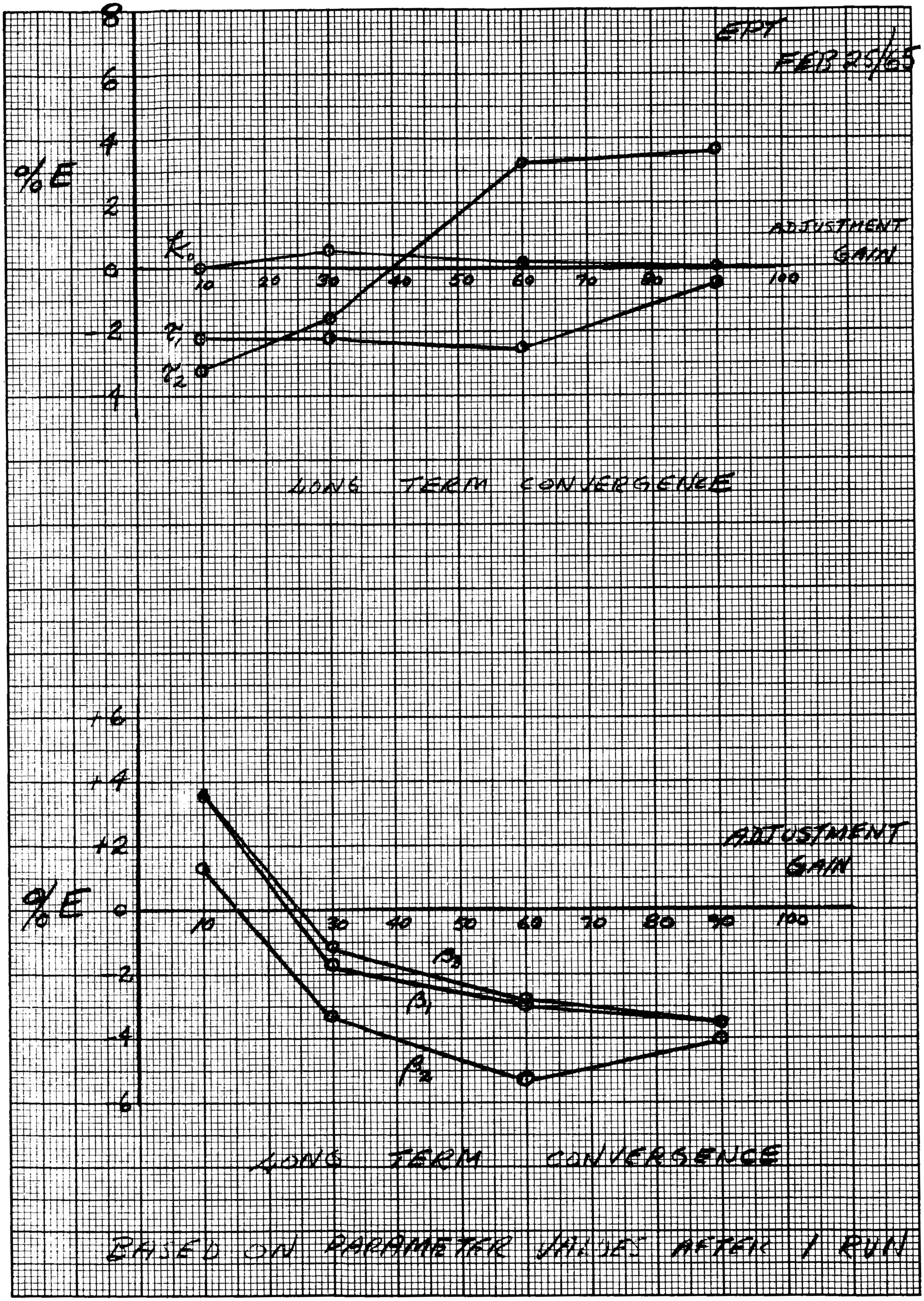

Figure 5. Effect of adjustment gain on long term convergence accuracy

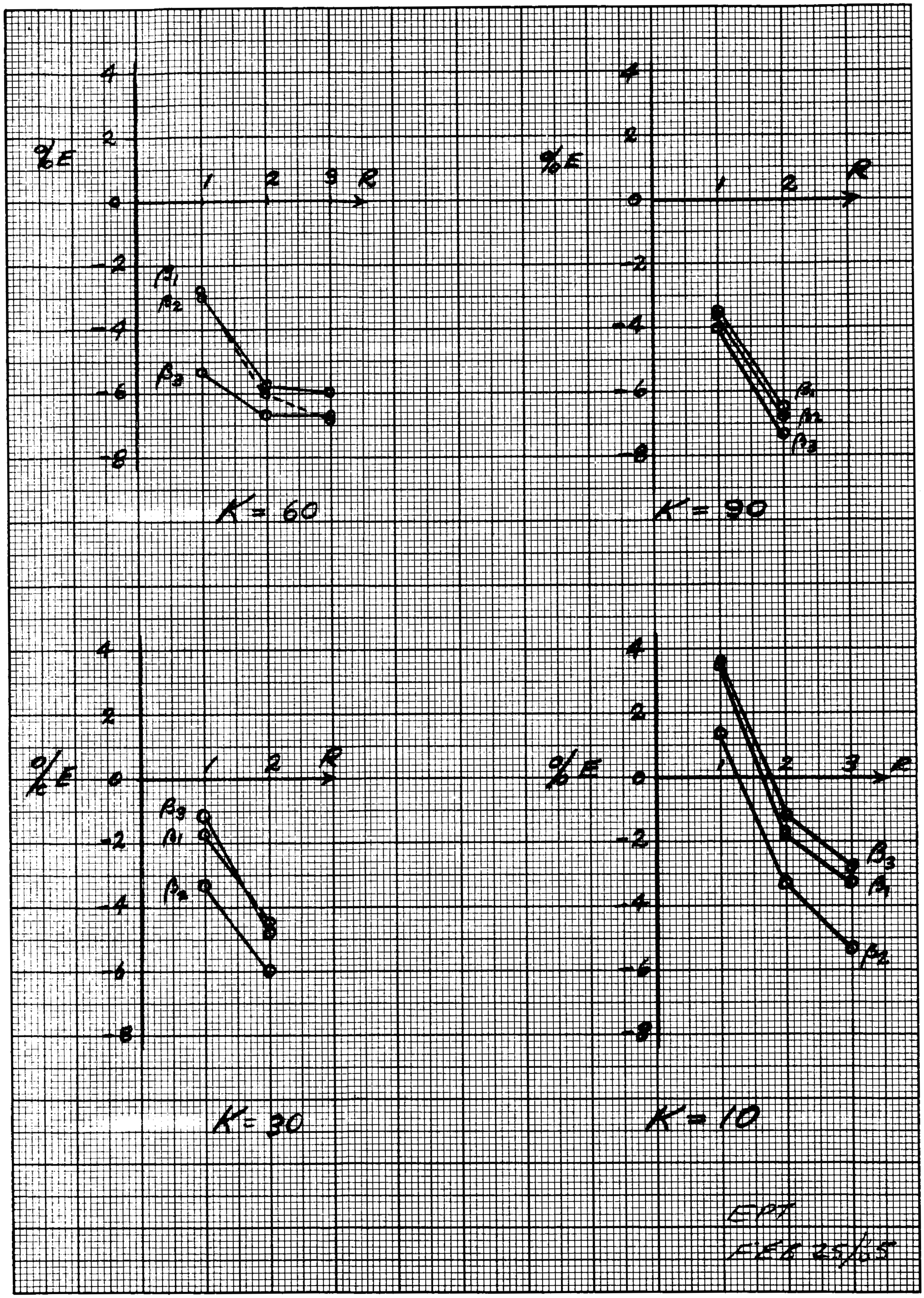

Figure 6. Effect of replication on long term convergence accuracy
(β parameters)

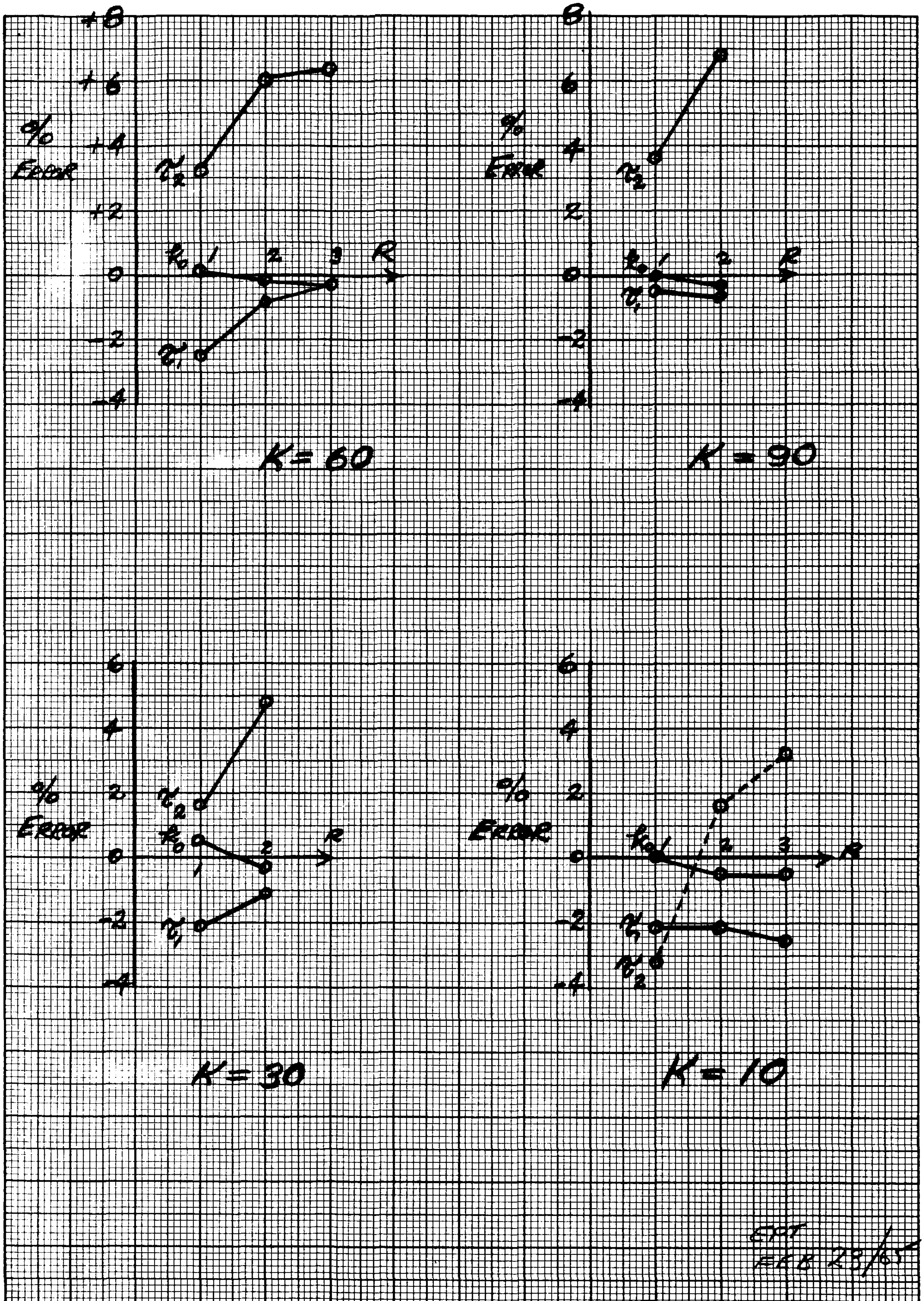

Fig. 7. Effect of replication on long term convergence accuracy (transfer function parameters)

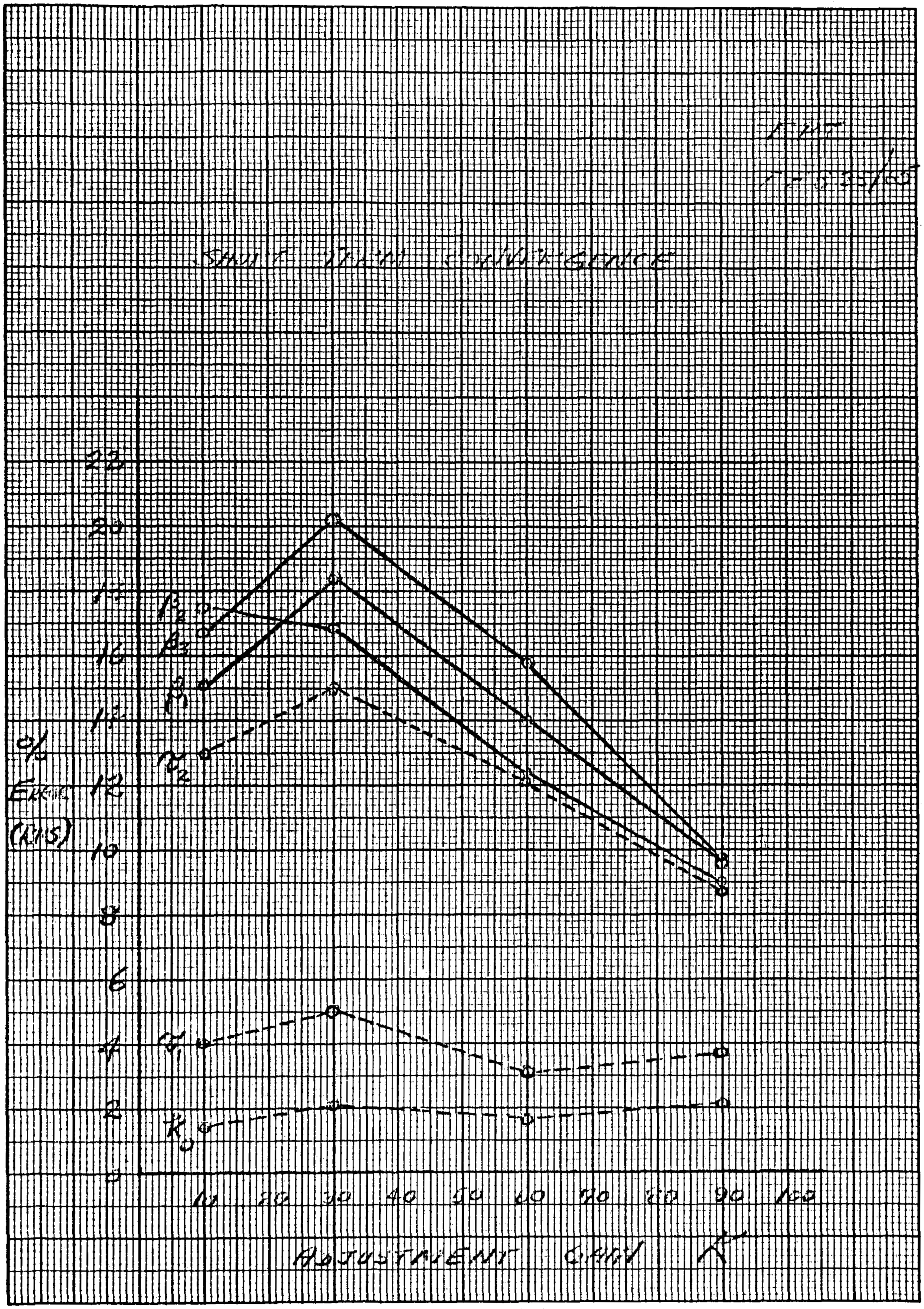

Fig. 8. Effect of adjustment gain on short term convergence accuracy

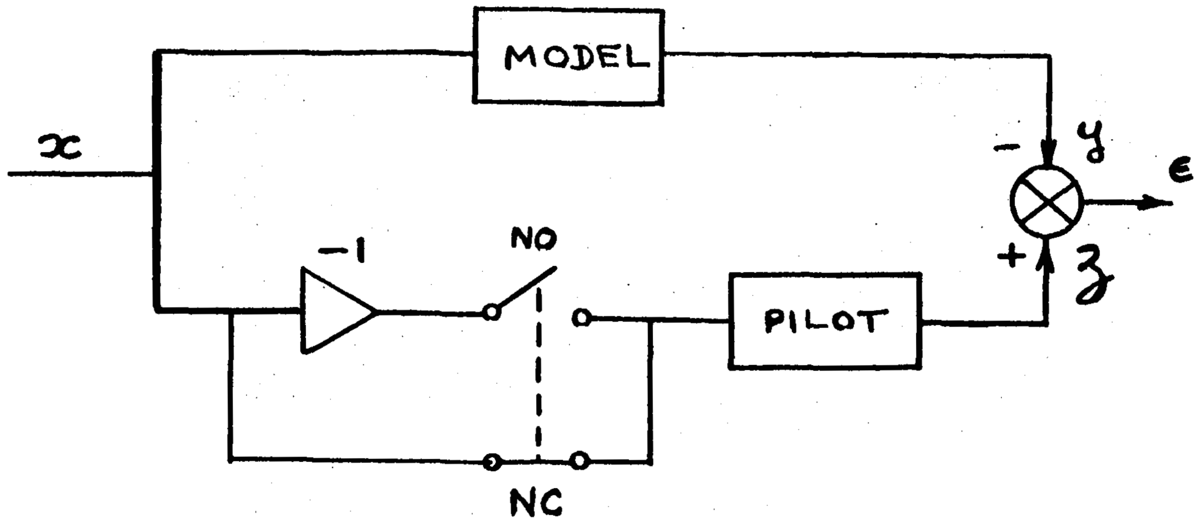

Figure 9. Control reversal simulation block diagram

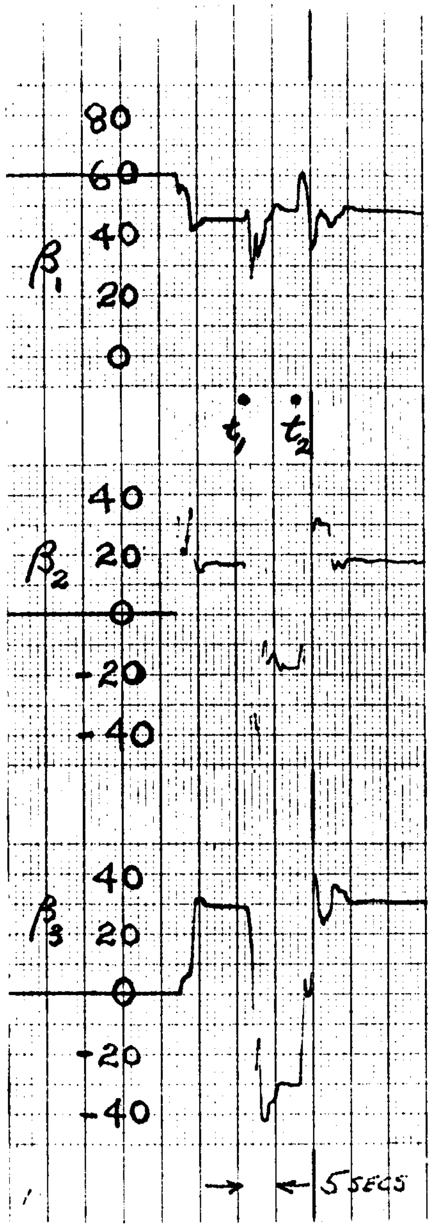

Figure 10. Simulation of control reversal

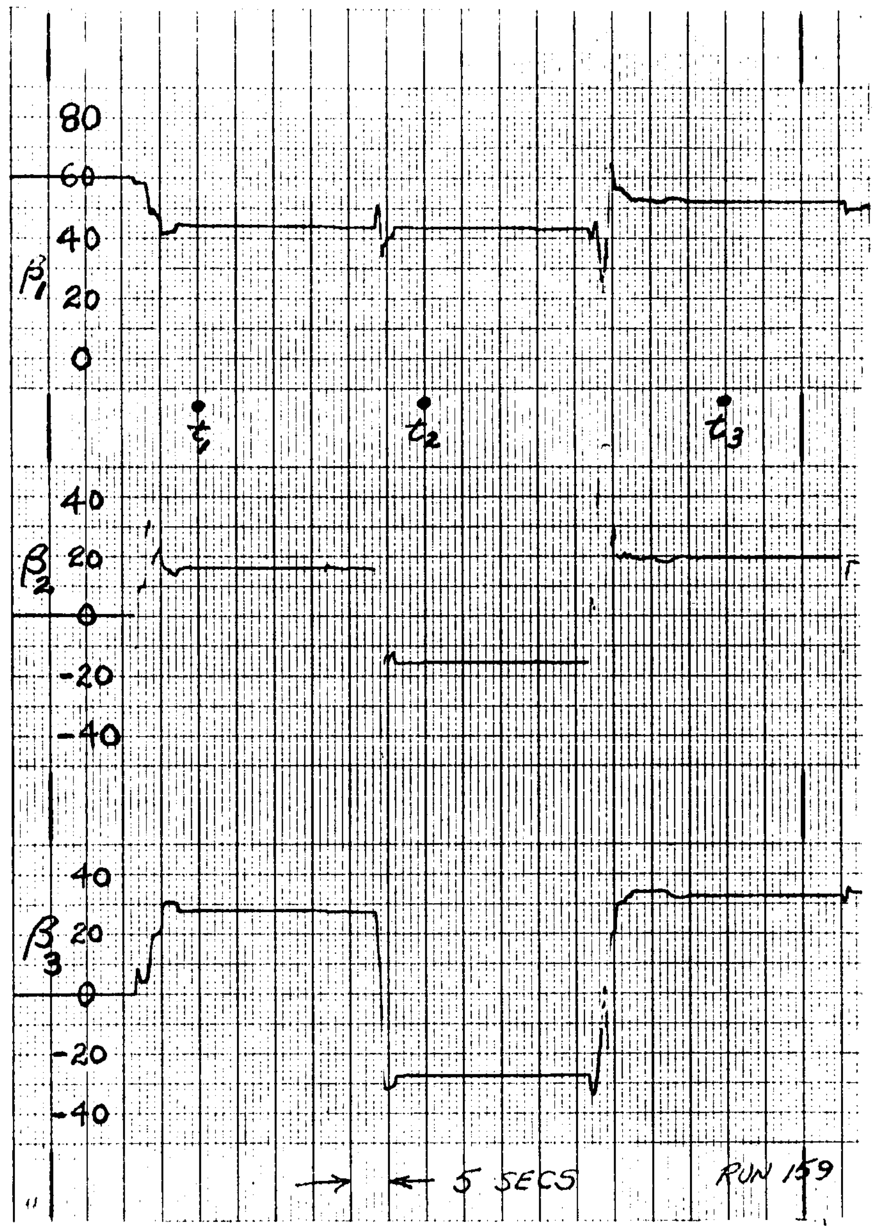

Figure 11. Parameter accuracy during the control reversal.

TRW SPACE TECHNOLOGY LABORATORIES
THOMPSON RAMO WOOLDRIDGE INC.

INTEROFFICE CORRESPONDENCE

TO: Distribution	CC:	DATE:	9350.6-153 26 February 1965
SUBJECT: Relative Sensitivity of Human Pilot Model Parameters		FROM:	H. F. Meissinger
	BLDG. R2	ROOM 1086	EXT. 22115

1. Problem Statement

Experimental results indicate that the parameters in a transfer function model of the human operator, especially the steady state gain factor K_0, tend to be determined with greater precision by the model matching process than the individual coefficients of an equivalent differential equation model. This result is traceable to the relative magnitude of the sensitivities of the various parameters.

The purpose of analyzing these relationships is to confirm the trends exhibited by the experimental results in quantitative and qualitative terms, and to find criteria for selecting mathematical model structures that yield to parameter identification processes with higher precision than others. On the basis of this analysis it will also be possible to distinguish between a case of poor computer accuracy and a mathematically unfavorable choice of the task which the computer is asked to perform.

The dissimilar relative accuracy levels for different model parameters are explained by the fact that the error term ϵ is in first approximation a weighted sum of the individual parameter errors $\Delta\alpha_i$,

$$\epsilon = \epsilon(0) + \sum_{i=1}^{n} u_i \Delta\alpha_i \tag{1}$$

where the u_i are the sensitivity coefficients of z with respect to α_i (see Reference 1, pages 50-72). If equal adjustment gain settings K are used in the different parameter adjustment circuits, the parameters α_j

having the largest relative sensitivity u_j will be defined with greater precision than others. In a two-parameter case, for example, the contours of the error criterion $F = \int e^2 \, dt$ will be elongated in the direction of the parameter having a weaker sensitivity (Reference 1, pages 56-58), the error criterion F being less sensitive in this direction to residual parameter errors.

2. Equivalent Model Forms

In this discussion we compare the first order linear model differential equation

$$\dot{z} + \beta_1 z = \beta_2 \dot{x} + \beta_3 x \tag{2}$$

having parameters β_i with the equivalent transfer function model

$$\frac{Z}{X} = K_0 \frac{\tau_1 s + 1}{\tau_2 s + 1} \tag{3}$$

where

$$K_0 = \frac{\beta_3}{\beta_1}, \qquad \tau_1 = \frac{\beta_2}{\beta_3}, \qquad \tau_2 = \frac{1}{\beta_1} \tag{4}$$

Both model forms (2) and (3) have been used interchangeably in previous work. The differential equation corresponding to (3)

$$\tau_2 \dot{z} + z = K_0 (\tau_1 \dot{x} + x) \tag{5}$$

is derived from (2) by multiplication with τ_2. Computer results (Reference 2) show that K_0 is a well-defined parameter, whereas the terms β_1, β_3 which determine K_0 tend to drift simultaneously or yield somewhat inconsistent results in repeated modeling runs of the same human operator tracking data. τ_1 and τ_2 are also defined with greater relative accuracy than the corresponding β_i terms.

3. Sensitivity Equations and Sensitivity Ratios

The influence coefficients $u_i = \partial z / \partial \beta_i$ are obtained by solution of the sensitivity equations derived from (2)

$$\begin{aligned} \dot{u}_1 + \beta_1 u_1 &= -z \\ \dot{u}_2 + \beta_1 u_2 &= \dot{x} \\ \dot{u}_3 + \beta_1 u_3 &= x \end{aligned} \tag{6}$$

Using the notation of (3) and (4) the equivalent transform equations

$$\begin{aligned} U_1 &= -K_0 \tau_2 \frac{\tau_1 s + 1}{(\tau_2 s + 1)^2} X \\ U_2 &= \frac{\tau_2 s}{\tau_2 s + 1} X = s U_3 \\ U_3 &= \frac{\tau_2}{\tau_2 s + 1} X \end{aligned} \tag{7}$$

are obtained, assuming zero initial values. Similarly the sensitivity equations for

$$v_0 = \frac{\partial z}{\partial K_0}, \quad v_1 = \frac{\partial z}{\partial \tau_1}, \quad v_2 = \frac{\partial z}{\partial \tau_2}$$

yield

$$\begin{aligned} V_0 &= \frac{\tau_1 s + 1}{\tau_2 s + 1} X \\ V_1 &= -K_0 \frac{s(\tau_1 s + 1)}{(\tau_2 s + 1)^2} X \\ V_2 &= K_0 \frac{s}{\tau_2 s + 1} X \end{aligned} \tag{8}$$

For simplification of the subsequent discussion we form the sensitivity ratios

$$\begin{aligned} q_{12} &= \frac{U_1}{U_2} = -\frac{K_0(\tau_1 s + 1)}{s(\tau_2 s + 1)} & r_{01} &= \frac{V_0}{V_1} = \frac{\tau_1 s + 1}{K_0 s} \\ q_{13} &= \frac{U_1}{U_3} = -\frac{K_0(\tau_1 s + 1)}{\tau_2 s + 1} & r_{02} &= \frac{V_0}{V_2} = -\frac{\tau_2 s + 1}{K_0 s} \\ q_{23} &= \frac{U_2}{U_3} = s & r_{12} &= \frac{V_1}{V_2} = -\frac{\tau_2 s + 1}{\tau_1 s + 1} \end{aligned} \tag{9}$$

These expressions which permit an estimate of the relative magnitude and power of the sensitivities u_i and v_i are illustrated by Bode graphs shown in Figures 1 and 2, respectively, for a typical case where the parameter values are

$$\beta_1 = 40\ sec^{-1} \qquad \qquad K_o = 0.625$$
$$\beta_2 = 15 \qquad \text{or} \qquad \tau_1 = 0.600\ sec$$
$$\beta_3 = 25\ sec^{-1} \qquad \qquad \tau_2 = 0.025\ sec$$

(This parameter condition has been the subject of an extensive experimental model matching study and data analysis as reported in Reference 2.) Table 1 gives the characteristics of the functions r and q used in constructing the Bode plots.

Table 1

Characteristics of Sensitivity Ratios

	G_0	G_∞
r_{01}	∞	$\frac{\tau_1}{K_0} = 0.96$
r_{02}	∞	$\frac{\tau_2}{K_0} = 0.040$
r_{12}	1	$\frac{\tau_2}{\tau_1} = 0.0416$
q_{12}	∞	0
q_{13}	K_0	$K_0 \frac{\tau_1}{\tau_2} = 15$
q_{23}	0	∞

While r and q give relative sensitivities of the parameters within the models (2), (3) respectively, the relative sensitivities between the models are expressed by the ratios $\frac{U_3}{V_2}$, $\frac{U_3}{V_1}$, $\frac{U_3}{V_0}$, etc. The term

$$\frac{U_3}{V_o} = \frac{\tau_2}{\tau_1 s + 1} \tag{10}$$

is plotted in Figure 2. Using this term for calibration the other relative inter-model sensitivities can be deduced.

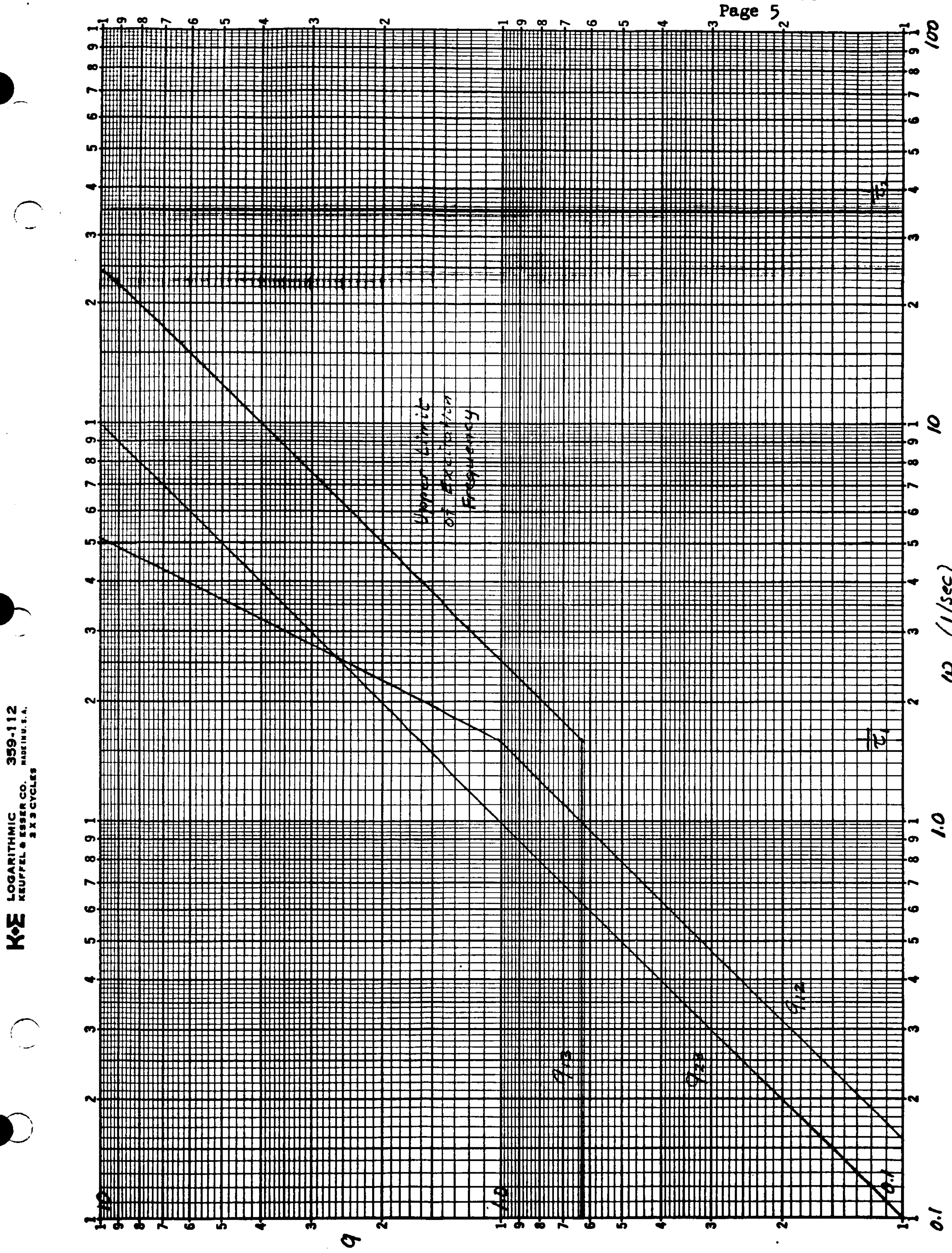

Figure 1. Frequency Dependence of Sensitivity Ratios q

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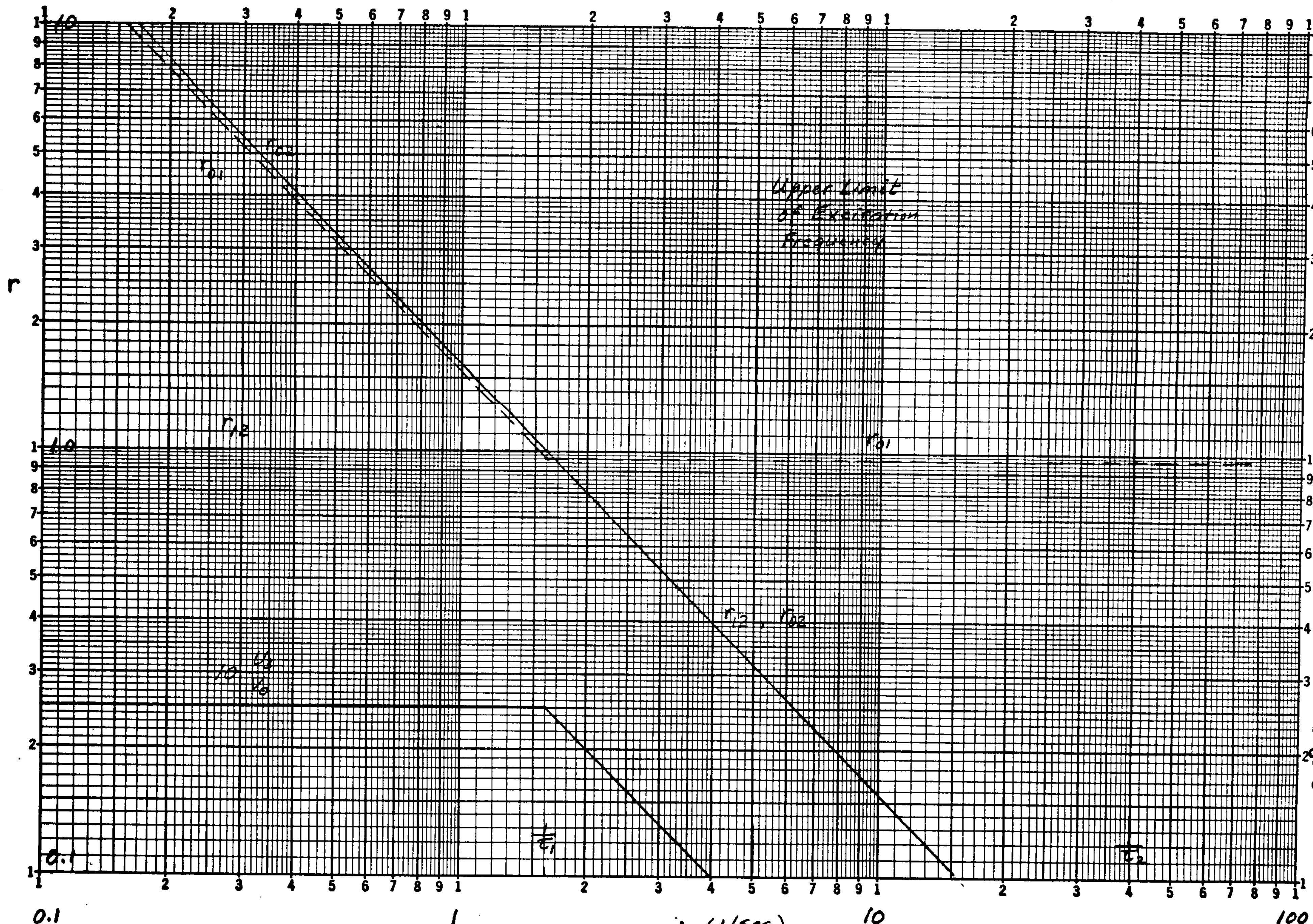

Figure 2. Frequency Dependence of Sensitivity Ratios r and U_3/V_0

4. Discussion

In evaluating the amplitude vs. frequency plots for r and q one must take into account the upper frequency limit of the excitation signal x(t) occurring in human tracking studies. On the basis of past experiments we set the cutoff frequency roughly at ω_c = 5 rad/sec to obtain estimates of relative magnitudes of the u_i and v_i. (The resulting estimates reflect this choice of ω_c.) In the frequency range of interest the sensitivity ratios behave as follows:

Table 2

Differential Equation Parameters	Transfer Function Parameters
$q_{12} \sim 0.1 \ldots 10$	$r_{01} \sim 15 \ldots 1$
$q_{13} \sim 0.6 \ldots 2$	$r_{02} \sim 15 \ldots 0.3$
$q_{23} \sim 0.1 \ldots 5$	$r_{12} \sim 1 \ldots 0.3$

Between Models

$$\left|\frac{V_0}{U_3}\right| \sim 40 \ldots 100$$

This leads to the following observations:

1. The parameters β_1, β_2, β_3 have essentially the same degree of sensitivity except that u_3 dominates u_2, u_1 in the lower frequency region, u_2 dominates u_1 at low frequencies, u_3 at high frequencies. On the average the sensitivities are approximately matched. This agrees with the findings, in Reference 2, of comparable accuracy of all β's.

2. The parameter sensitivities for K_0, τ_1, τ_2 show larger dispersions. v_0 dominates v_1 and v_2 very distinctly up to frequencies of 1.5 rad/sec. v_1 and v_2 are of similar magnitude, but v_2 tends to dominate v_0 and v_1 in the upper frequency range. The high

accuracy of K_0 exhibited in the experimental study confirms this result. On the other hand τ_2 shows consistently poor accuracy compared to τ_1 and K_0, but should be expected to be at least on a par with τ_1. This discrepancy points to possible computational inaccuracy in the results of Reference 2.

3. The most striking difference in sensitivities is indicated by the behavior of V_0/U_3. Thus for the case investigated the steady state gain K_0 is determined with an accuracy at least an order of magnitude higher than the parameters β_i. In view of the values r_{01}, r_{02} and the ratio V_0/U_3 we deduce that τ_1 and τ_2 should also be considerably more well defined than the β_i's. This finding is confirmed by the experimental results.

   Additional insight is gained by noting that

$$\frac{V_1}{U_3} = \frac{K_0}{\tau_2} s = 25 s$$

$$\frac{V_2}{U_1} = \frac{s}{\tau_2} = 40 s$$

   which shows the dominance of v_1 and v_2 over the u's.

4. The above results are largely parameter-dependent. For example, r_{01} is shaped by τ_1 and K_0. Figure 3 illustrates how r_{01} varies with increases in each of these parameters. As τ_1 increases, the dominance of v_0 is enhanced, an increase in K_0 has the opposite effect. The dominance of v_0 over u_1, u_2, u_3 depends strongly on τ_2. For increased τ_2 (human pilot lag time constant) to more typical values of 0.1 - 0.2 sec the preponderance of v_0 decreases by an order of magnitude but is still noticeable. τ_1 has a much smaller effect on the ratio U_3/V_0 unless τ_1 is substantially increased above the 0.6 sec value used in this discussion.

   r_{12} and q_{23} are largely uninfluenced by parameter changes.

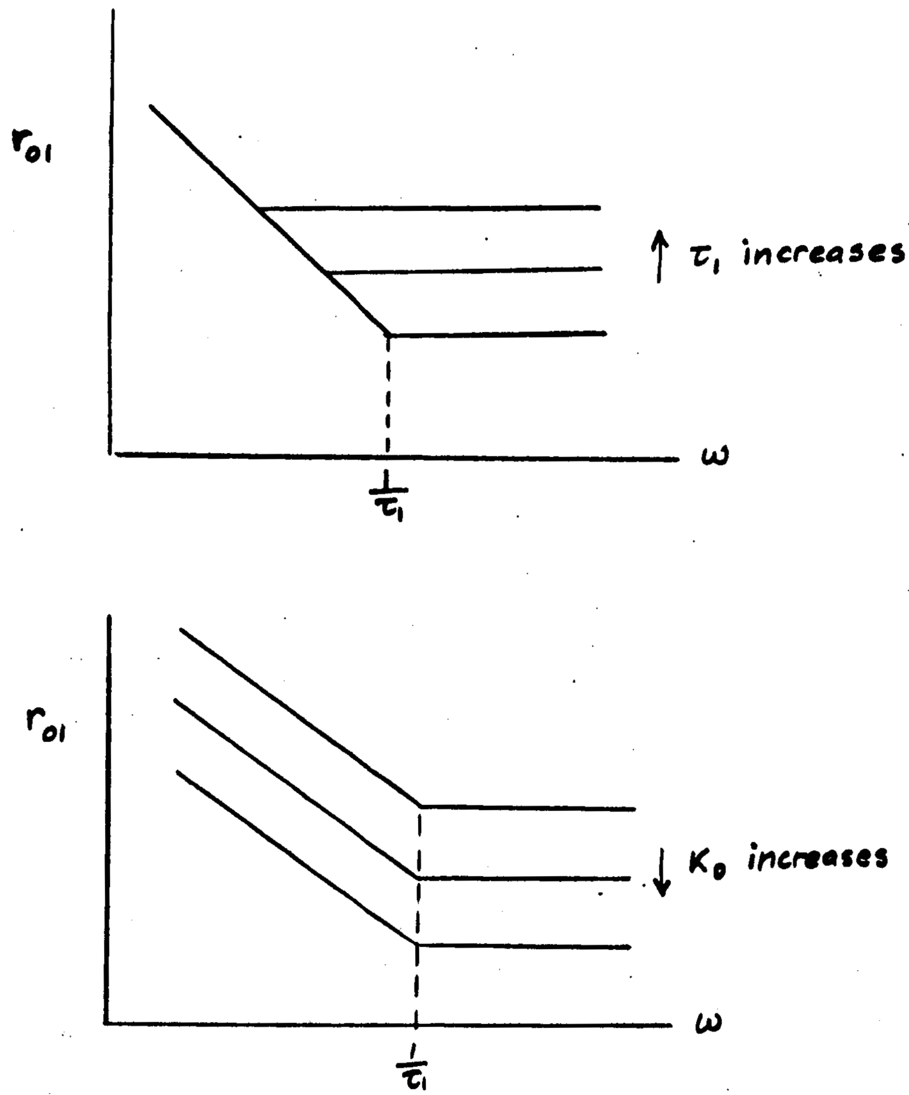

Figure 3. Parameter Dependence of r_{01}

5. Conclusion

The simple analytical method presented here is very useful in detecting sources of parameter definition accuracy or inaccuracy which may otherwise remain obscure. The method can be readily extended to practical problems characterised by second order models, but remains limited to linear structures.

The method serves to pinpoint mathematically favorable model formats or parameter combinations to be selected for the optimization program. As a general method of sensitivity analysis it has a range of applications in control engineering, system optimization, adaptive control, and related fields where it should be further pursued.

H. F. Meissinger

H. F. Meissinger

HFM:js

Distribution

G. A. Bekey
W. A. Finley
R. E. Rose
L. G. Summers
E. P. Todosiev

References:

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TRW SPACE TECHNOLOGY LABORATORIES
THOMPSON RAMO WOOLDRIDGE INC.

INTEROFFICE CORRESPONDENCE

TO: Distribution CC: DATE: 9350.6-152
26 February 1965

SUBJECT: Analysis of Time Delay Approximation in Human Pilot Models

H. F. Meissinger

FROM: H. F. Meissinger
BLDG. R2 ROOM 1086 EXT. 22115

1. Method of Approximation

A first order approximation of the effect of time delay τ in the mathematical model of a human operator can be obtained without actual implementation of a time delay term in the computer circuits (see Reference 1, pg. A-23). The time-delayed response is obtained by extrapolation from the solution for $\tau = 0$, using first order parameter influence terms.

Consider for example the model equation with time delay in the input signal

$$\ddot{z} + \alpha_1 \dot{z} + \alpha_2 z = \alpha_3 x(t-\tau) \tag{1}$$

where

$x(t)$ = input signal to human operator

$x(t-\tau)$ = time delayed input signal

z = output of the mathematical pilot model

$\alpha_1, \alpha_2, \alpha_3$ = model parameters.

A first order extrapolation in the vicinity of $\tau = 0$

$$z_1(t;\tau) = z_0 + \frac{\partial z_0}{\partial \tau}\tau \tag{2}$$

(where $z_0 = z_{\tau=0}$) is obtained from

$$\ddot{z}_0 + \alpha_1 \dot{z}_0 + \alpha_2 z_0 = \alpha_3 x(t) \tag{3}$$

and the partial derivative of z_0 with respect to τ.

This derivative, the influence coefficient $u_\tau = \frac{\partial z_0}{\partial \tau}$, is obtained by solution of the sensitivity equation

$$\ddot{u}_\tau + \alpha_1 \dot{u}_\tau + \alpha_2 u_\tau = -\alpha_3 \dot{x}(t) \tag{4}$$

Eq. (4) is derived from (3) by observing that

$$\frac{\partial x(t-\tau)}{\partial \tau} = -\frac{dx}{dt} = -\dot{x}(t) \tag{5}$$

at $\tau = 0$. In the vicinity of $\tau = 0$ equation (5) is an acceptable approximation.

Equations (3) and (4) yield

$$u_\tau \simeq -\dot{z}_0 \tag{6}$$

u_τ equals $-\dot{z}_0$ only if the initial conditions of the respective differential equations (4) and (3) are the same. Since for purposes of this study the model output is determined by the continuing random excitation signal $x(t)$ the importance of initial values in (3) and (4) is minimized, provided the homogeneous solutions are damped. Substitution of (6) in (2) yields

$$z_1 = z_0 - \dot{z}_0 \tau \tag{7}$$

This could also have been derived directly in terms of operational calculus, using the first order term in the series

$$e^{-\tau s} = 1 - \tau s + \frac{1}{2!}\tau^2 s^2 \ldots \tag{8}$$

2. Analysis of Approximation Error

a) Sinusoidal Excitation

In the case of a purely sinusoidal input

$$x = A \sin \omega t = A \sin \theta \tag{9}$$

$$x(t-\tau) = A \sin\omega(t-\tau) = A\sin(\theta-\lambda) \tag{10}$$

where $\omega t = \theta$, $\omega\tau = \lambda$

the approximation error can be explicitly derived as follows. Let the steady state solutions z, z_0, $\dot{z}_0$ be denoted by

$$\left.\begin{aligned} z &= B\sin(\theta-\lambda-\varphi) \\ z_0 &= B\sin(\theta-\varphi) \\ \dot{z}_0 &= B\omega\cos(\theta-\varphi) \end{aligned}\right\} \tag{11}$$

The approximate model output is

$$z_1 = z_0 - \dot{z}_0\tau = B\left[\sin(\theta-\varphi) - \lambda\cos(\theta-\varphi)\right] \tag{12}$$

Thus the approximation error is

$$\epsilon_1 = z - z_1 = B\left[\sin(\theta-\lambda-\varphi) - \sin(\theta-\varphi) + \lambda\cos(\theta-\varphi)\right] \tag{13}$$

To further simplify the notation, let $\theta - \lambda = \psi$. Thus from (13)

$$\epsilon_1 = B\left[\sin\psi(\cos\lambda - 1) + \cos\psi(\lambda - \sin\lambda)\right] \tag{14}$$

Zero error occurs at times when

$$\tan\psi_1 = \frac{\lambda - \sin\lambda}{1-\cos\lambda} \tag{15}$$

$$\tan\psi_1 \simeq \frac{\lambda}{3} \qquad \ldots \text{ for small } \lambda$$

The maxima occur when $\frac{d\epsilon}{d\psi} = 0$, i.e., for

$$\tan\psi_2 = -\frac{1-\cos\lambda}{\lambda-\sin\lambda} \tag{16}$$

$$\tan\psi_2 \simeq -\frac{3}{\lambda} \qquad \ldots \text{ for small } \lambda$$

As illustrated in Figure 1 the approximation error ϵ_1 is nearly zero at the nulls of z_0 and maximum a small time interval after the maxima of z_0. The nulls and maxima of ϵ_1 are shifted from the nulls and maxima of z_0 by the small phase angle ϕ_1.

It should be noted that these assumptions do not presuppose a model equation of the form (1). Higher order linear differential equations in z and higher order terms in $x(t - \tau)$ yield the same result.

The maximum error $E = \epsilon_{1\ max}$ is obtained by substituting (16) into (14).

$$E = B\sqrt{(\cos\lambda - 1)^2 + (\lambda - \sin\lambda)^2}$$

$$E \simeq \frac{1}{6} B\lambda^2 \sqrt{\lambda^2 + 9} \quad \text{... for small } \lambda \tag{17}$$

The results are tabulated in Table 1 for λ values ≤ 1.

Table 1

Normalized Maximum Error E

$\lambda = \omega\tau$ (rad)	E/B
0	0
0.2	0.020
0.4	0.081
0.6	0.184
0.8	0.313
1.0	0.489

Figure 2 shows graphs of the normalized maximum error as function of time delay τ for various values of the excitation frequency ω. For the frequency range of interest ($\omega \leq 1$ rad/sec) and typical human pilot time delay ($\tau \leq 0.3$ sec) the error is considerably smaller than 5 percent of the maximum model output.

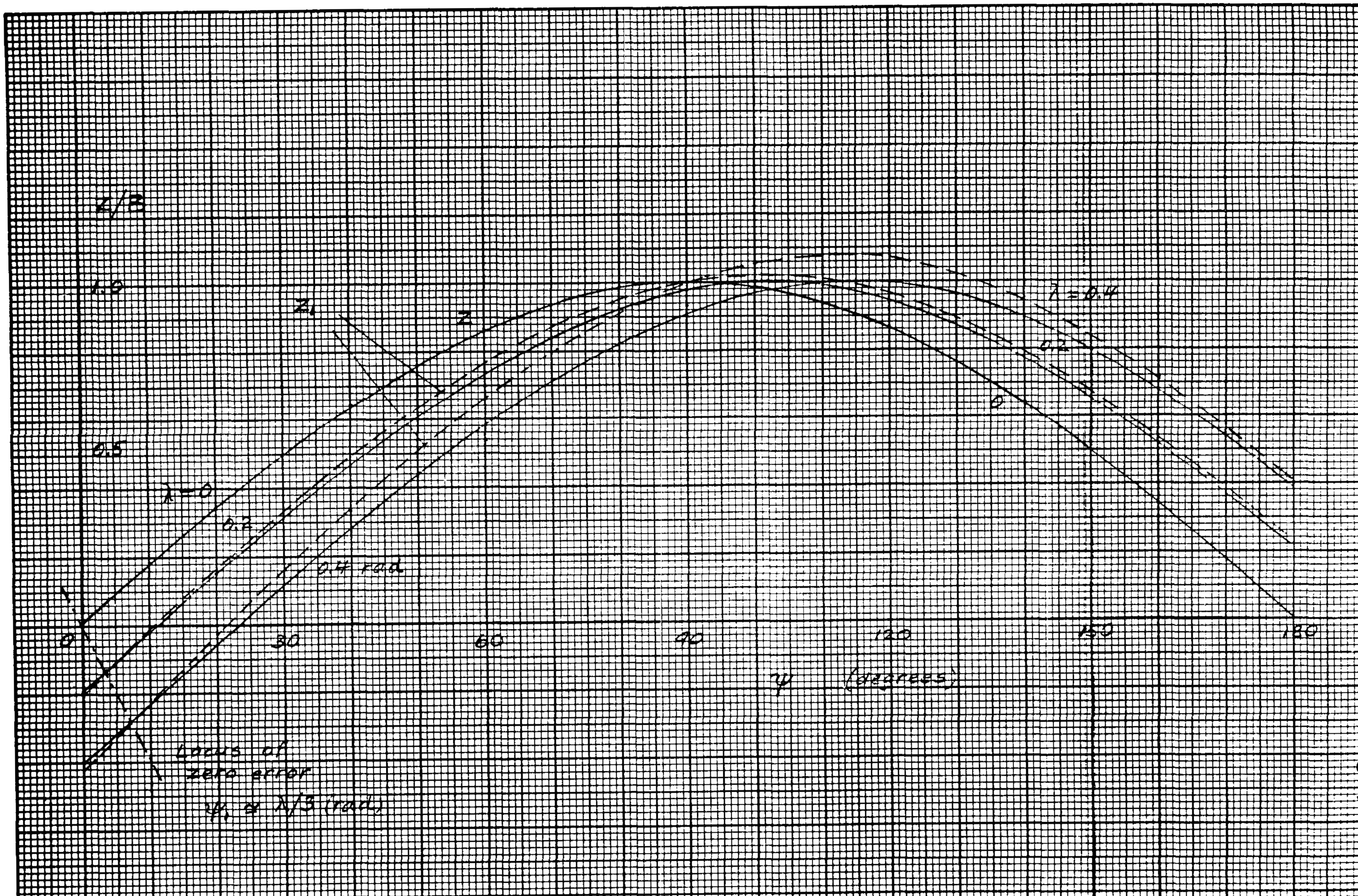

Figure 1. Time-Delayed Response z and Approximate Response z_1 vs. Time

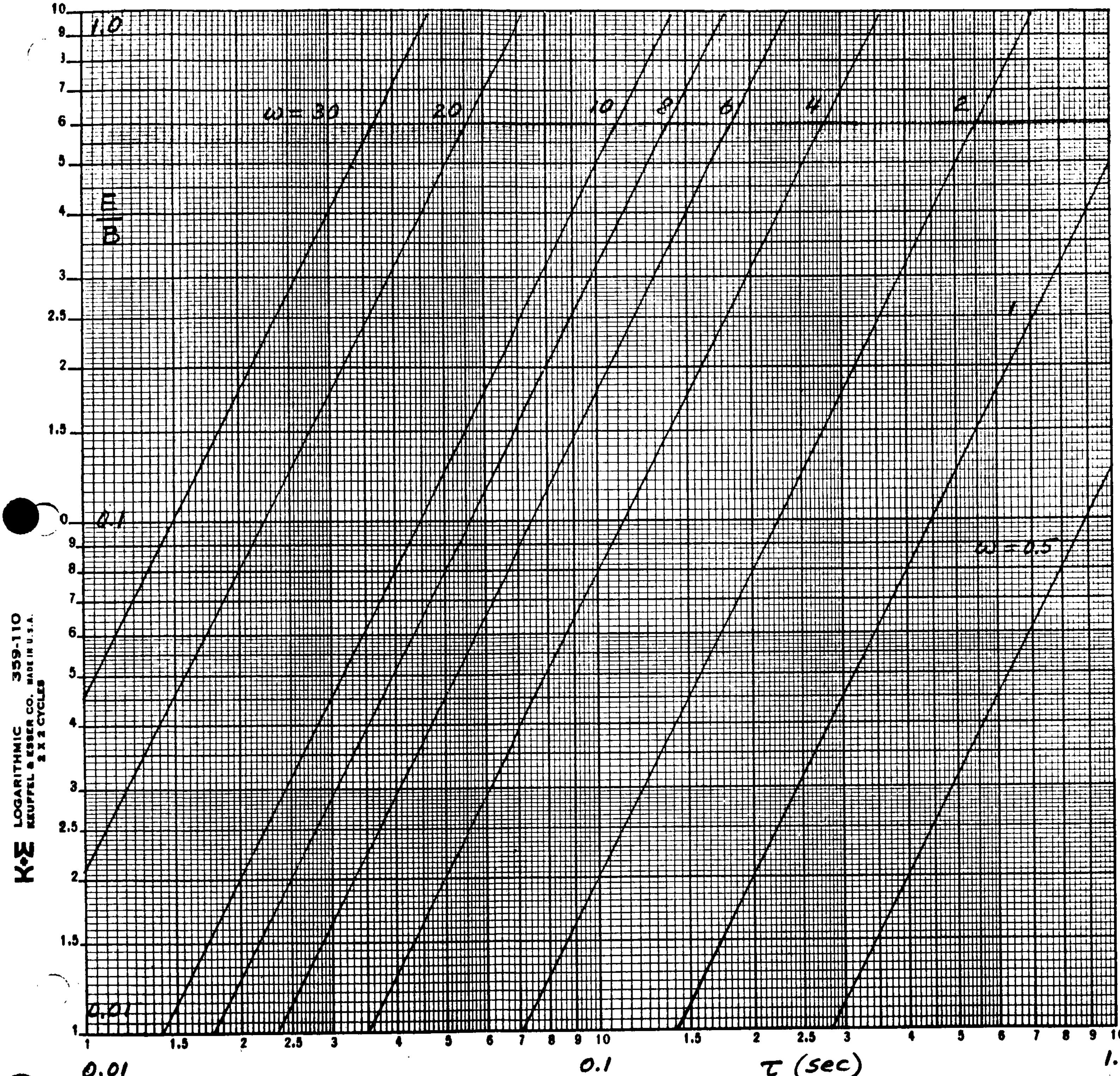

Figure 2. Normalized Maximum Error vs. Time Dalay τ

b) Random Excitation

For random excitation the above results provide estimates of the amplitude distortion at various frequencies in the model response. For excitation bandwidths of approximately 1.0 rad/sec only the upper frequency components exhibit noticeable distortions on the order of 5 percent.

3. Prediction of Time Delay by Steepest Descent Adjustment of τ

If the approximate model output z_1 is used as basis of a computer prediction of τ (see Reference 1, pg.A-23)a prediction error must be expected due to non-zero ϵ_1 values. Let the predicted value be τ_1.

The steepest descent equation for determination of τ_1 for model matching of z by means of z_1 is of the form

$$\dot{\tau}_1 = -K \frac{\partial f}{\partial \tau_1} = -K \epsilon_1 \frac{\partial \epsilon_1}{\partial \tau_1} \tag{18}$$

where

$$f = \frac{1}{2} \epsilon_1^2$$

With

$$\epsilon_1 = z - z_1 = z - z_o + \dot{z}_o \tau_1$$

one obtains

$$\frac{\partial \epsilon_1}{\partial \tau_1} = \dot{z}_o \tag{19}$$

$$\dot{\tau}_1 = -K \epsilon_1 \dot{z}_o \tag{20}$$

Note that a computer circuit for obtaining τ_1 estimates can be set up without requiring a time delay unit, simply by using the available terms ϵ_1 (model matching error) and $\dot{z}_0$, as shown in Figure 3.

Theoretically the computer-predicted time delay τ_1 is obtained as follows. For sinusoidal excitation $x(t)$ at frequency ω equation (20) may be expressed by

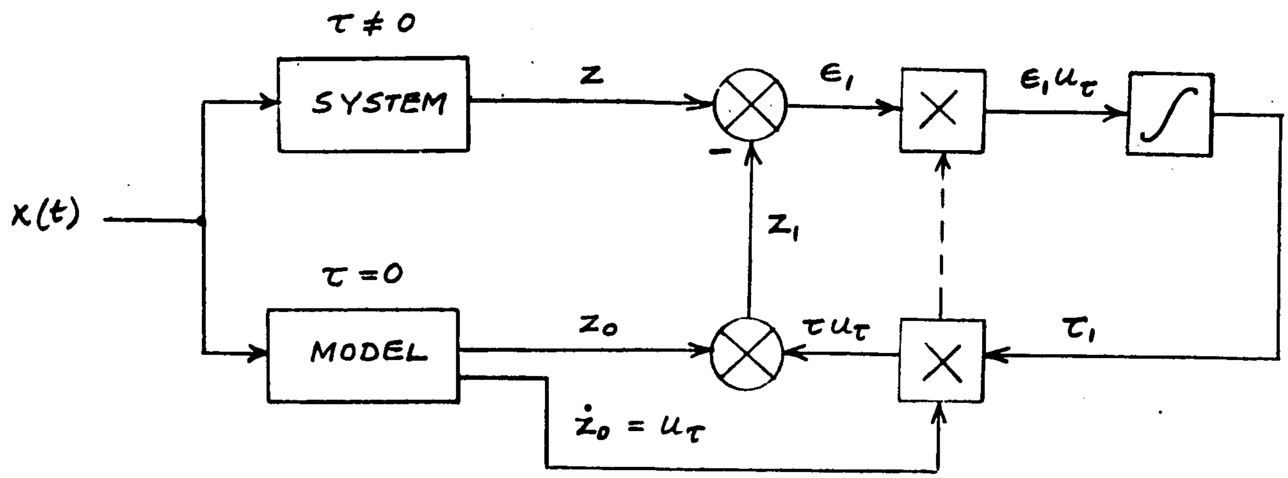

Figure 3. Computer Circuit for Determination of τ_1

$$\dot{\lambda}_1 = -K\epsilon_1\omega\dot{z}_o = -KB\omega\dot{z}_o\left[\sin(\psi-\lambda) - \sin\psi + \lambda_1\cos\psi\right] \quad (21)$$

The error term ϵ_1 is sinusoidal with amplitude E_1

where

$$E_1 = B\sqrt{(\cos\lambda - 1)^2 + (\lambda_1 - \sin\lambda)^2} \quad (22)$$

similar to (17). Obviously, minimization of E_1^2 with respect to λ_1 by steepest descent requires

$$\lambda_1 = \sin\lambda \quad (23)$$

since

$$\frac{\partial E_1^2}{\partial \lambda_1} = 2B^2(\lambda_1 - \sin\lambda)$$

Therefore

$$E_{1\,min}^{\;2} = B^2(\cos\lambda - 1)^2 \quad (24)$$

The prediction error in λ and the residual error criterion amplitude $E_{1\,min}^{\;2}$ are listed in Table 2 and plotted in Figure 4. For ω and τ values of interest the λ-error is less than 0.01 radians. For random

Table 2

Approximation Error in λ

$\lambda = \omega\tau$ (deg)	(rad)	λ_1 (rad)	$\lambda - \lambda_1$ (rad)	$\frac{E_{1\,min}}{B}$
0	0	0	0	0
10	.1745	.1740	0.0005	0.015
20	.349	.342	0.007	0.060
30	.523	.500	0.023	0.134
40	.698	.642	0.056	0.233
45	.785	.707	0.078	0.293

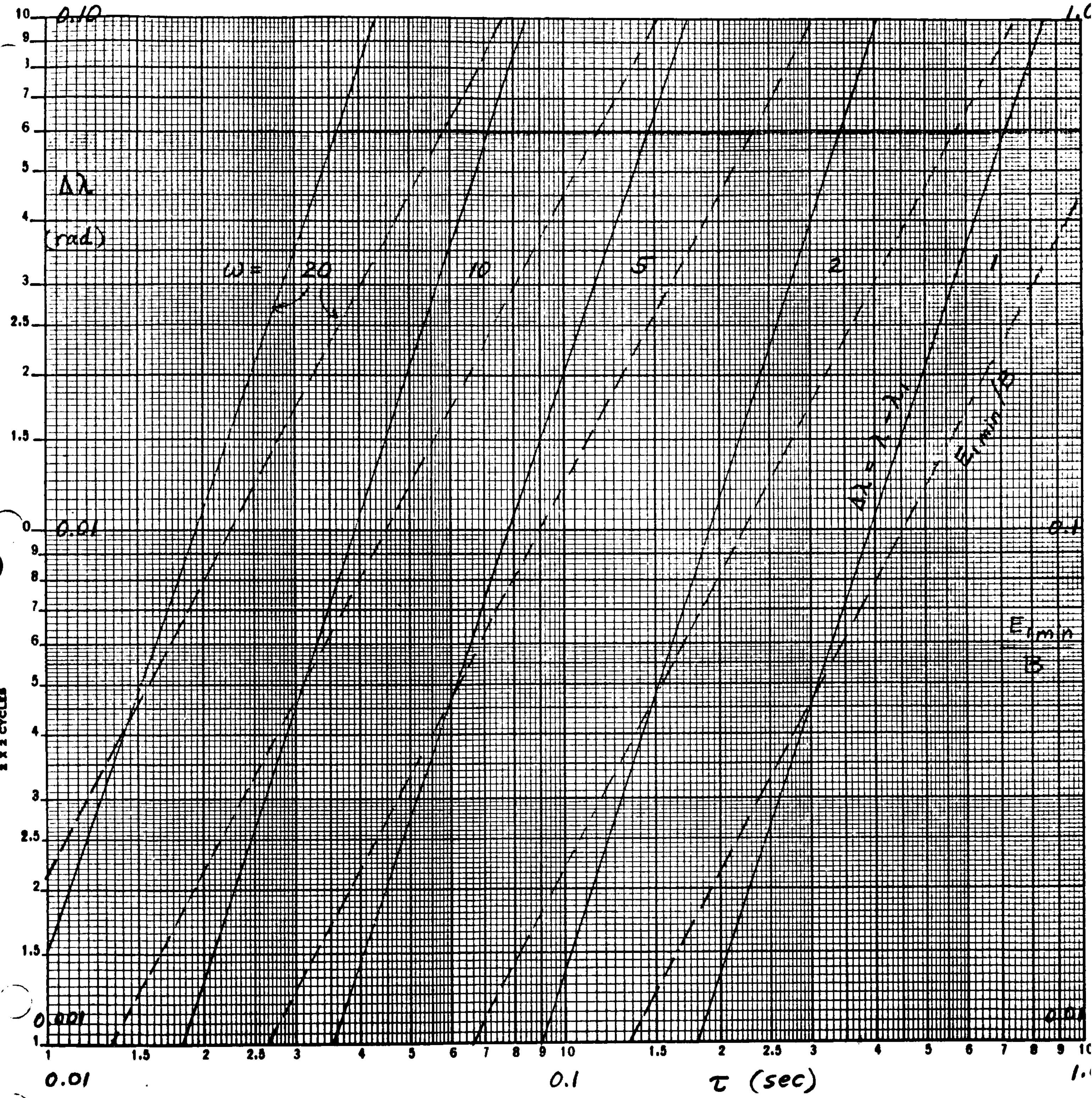

Figure 4. λ-Prediction Error and Residual Error Amplitude vs. Time Delay τ

excitation signals these results provide upper bounds on the estimation error in τ if the maximum frequency of the excitation signal is used in the graph.

4. Discussion

The foregoing analysis provides insight into the character of time delay approximation results obtainable by the proposed method, and establishes that approximation error is insignificant for frequencies and time delay values of practical interest in human pilot models. For system identification problems with much larger time-delay (e.g., process control) the above assumptions must be re-examined. But since perturbation frequencies are usually small in such cases the approximation is probably still quite useful. On the basis of these results the computer program depicted in Figure 3 will be utilized in Task 2, etc., of the subject study.

In the "non-ideal" case where the model structure and the system structure are dissimilar in terms other than the time delay τ, or if additional parameters are unmatched, the above discussion is no longer exact. The results only serve as reference data for the simple case of unmatched τ-terms.

R. Bellman (Reference 2) and other investigators have questioned the feasibility of imbedding the solution for $\tau = 0$ within a family of solutions for $\tau \neq 0$ under more general conditions, and point out fundamental difficulties of defining sensitivity here. The objective of the present analysis has been to establish feasibility and confidence levels for the case of interest in human pilot modeling as exemplified by equation (1).

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1. "Mathematical Models of Human Pilot Responses," STL Proposal 3667.000, dated 11 June 1964, pp. A-23 to A-25.

2. "Perturbation Methods in Science and Engineering," by R. Bellman, McGraw-Hill, 1964.

DISTRIBUTION

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INTEROFFICE CORRESPONDENCE

9350.6-156

TO: Distribution CC: DATE: 18 March 1965

SUBJECT: Approximation of Small Parameters Associated with Higher Order Terms in Human Pilot Models

FROM: H. F. Meissinger

BLDG. R2 ROOM 1086 EXT. 22115

The sensitivity of dynamic systems to small parameters that change the order of the mathematical model has been the subject of extensive studies by various investigators. Difficulties arise due to the fact that in general a singular condition exists at the time $t = 0$ such that the parameter sensitivity of the solution cannot be rigorously defined at this point.

The purpose of this analysis is to show that this problem is of no serious practical concern for purposes of human pilot modeling. A simple computation program can be formulated to obtain the values of small parameters of this type which have been omitted in the format of the model equation. Estimates for the errors inherent in the method are derived.

H. F. Meissinger

H. F. Meissinger

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APPROXIMATION OF SMALL PARAMETERS ASSOCIATED WITH HIGHER ORDER TERMS IN HUMAN PILOT MODELS

1. PROBLEM STATEMENT

The model differential equations used to represent the human pilot are usually assumed to be of first or second order. Small parameters which are associated with higher order terms can have a significant effect on the model response. Such effects may have to be investigated to determine model validity and accuracy.

Consider the example of a first order model

$$\dot{z} + \beta_1 z = \beta_2 \, x\,(t) \tag{1}$$

of a human pilot in comparison with a second order model

$$\lambda \ddot{z} + \dot{z} + \beta_1 z = \beta_2 \, x\,(t) \tag{2}$$

where λ is a small non-zero parameter. The transition from (1) to (2) introduces theoretical and practical computation difficulties which are the subject of this analysis.

The proposed method for approximating the effect of the λ-term on the model matching error ϵ (see Reference 1, p. A-22, A-23) is based on extrapolation in the vicinity of the solution z_0 obtained for $\lambda = 0$ using the first order parameter sensitivity coefficient of z_0, viz.,

$$z_1\,(t\,;\lambda) = z_0(t) + \frac{\partial z_0}{\partial \lambda}\,\lambda \tag{3}$$

provided such a sensitivity coefficient can be obtained. The effect of λ on the modeling error $\epsilon = z - y$ is approximated by

$$\epsilon_1\,(t\,;\lambda) = z_1\,(t\,;\lambda) - y\,(t)$$

$$= \epsilon_0\,(t) + \frac{\partial z_0}{\partial \lambda}\,\lambda \tag{4}$$

This method can be used to estimate the omitted term λ by steepest descent parameter optimization using the adjustment equation

$$\dot{\lambda} = -K \frac{\partial f}{\partial \lambda} = -K \epsilon \frac{\partial \epsilon}{\partial \lambda} = -K \epsilon u_\lambda \tag{5}$$

where $f = \frac{1}{2} \epsilon^2$ is the error criterion, $\frac{\partial f}{\partial \lambda} = \epsilon\, u_\lambda$ is the gradient component with respect to the parameter λ, and $u_\lambda = \frac{\partial z_0}{\partial \lambda} = \frac{\partial \epsilon_0}{\partial \lambda}$ denotes the influence coefficient. An analogous method for evaluating the effect on ϵ of time-delay terms has been proposed and analyzed in Reference 2.

Formally, the influence coefficient u_λ is obtained by solving the sensitivity equation of z_0 with respect to λ which is derived from (2)

$$\dot{u}_\lambda + \beta_1 u_\lambda = -\ddot{z}_0 \tag{6}$$

for the case $\lambda = 0$.

The mathematical problem arising in this approach centers on the initial conditions applying to equations (1) and (2). An arbitrary choice of $\dot{z}\,(0)$ in (2) does not permit a continuous variation of λ from non-zero values to $\lambda = 0$. The case $\lambda = 0$ is singular. Hence at $\underline{t = 0}$ the derivative $\frac{\partial z}{\partial \lambda}$ is not defined unless the initial value $\dot{z}\,(0)$ is chosen specifically so as to satisfy equation (1), viz.

$$\dot{z}(0) = \beta_2\, x\,(0) - \beta_1\, z\,(0) \tag{7}$$

This problem has been treated extensively in the literature on system sensitivity (e.g., References 3, 4) from a theoretical standpoint.

However, for purposes of model matching where the solution at times $t \neq 0$ is essentially determined only by the response to continuous random inputs rather than by the initial values of z, $\dot{z}$, ... the singularity at $t = 0$ is of no practical significance (see Reference 4). It is presupposed that we are dealing with stable systems where the effect of the initial state subsides rapidly.

This analysis will be concerned with the error inherent in the approximation (3) and with the accuracy of λ-values obtained by the solution of (5).

2. COMPUTATIONAL METHOD

The parameter adjustment program to be used in approximating λ by the steepest descent technique is illustrated in Figure 1. The sensitivity equation (6) requires the term $-\ddot{z}_0$ as driving function which is obtained by differentiation of $\dot{z}_0$. This term is available in the circuits representing the model equation. The differentiation requirement cannot be circumvented.

An alternative scheme can be devised which contains a term $\lambda\ddot{z}$ in the model equation thus solving (2) rather than (1) as shown in Figure 2. The λ-term can be obtained, as before, by model matching adjustment in accordance with (5). However, the introduction of the feedback term $\lambda\ddot{z}$ generated by explicit differentiation has caused difficulties in practical computer operation. Direct solution of (2) by more conventional programming is unsuitable for cases where λ approaches zero (see also Reference 5).

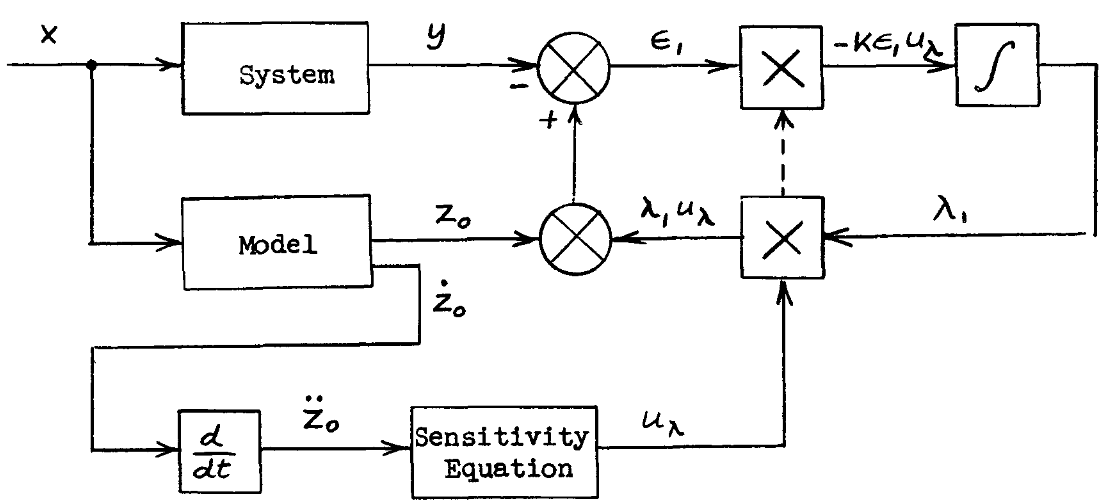


Figure 1. Computer Program for Determination of λ_1

A third scheme (Figure 3) has been devised which generates $\ddot{z}$ by implicit differentiation. This program is based on equation (2) rewritten in the form

$$\dddot{z} + (\lambda - 1)\ddot{z} + \dot{z} + \beta_1 z = \beta_2 x(t) \tag{2a}$$

The amplifier generating $\ddot{z}$ becomes in effect a high gain amplifier for $\lambda = 0$. Therefore the combination of amplifiers 1, 2 and integrator 3 performs the function of an implicit differentiator. It is noted that the transition from zero to finite values of λ is automatic when the human pilot response requires representation by the higher order model equation.

Corresponding computation schemes apply to transition from n^{th} to $(n+1)^{st}$ order models.

The proposed computation process, Figure 1, permits the evaluation of the effect non-zero λ values on the matching error ϵ without physically including a λ feedback circuit. As discussed in Reference 1 the λ-adjustment can be performed in open-loop manner and the approximate result λ_1 introduced into the error term, viz.

$$\epsilon_1 = \epsilon_0 + \lambda_1 u_\lambda$$

for observation of potential accuracy improvements.

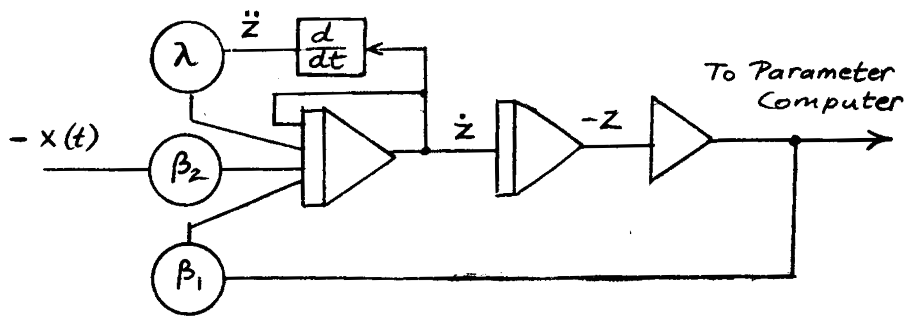


Figure 2

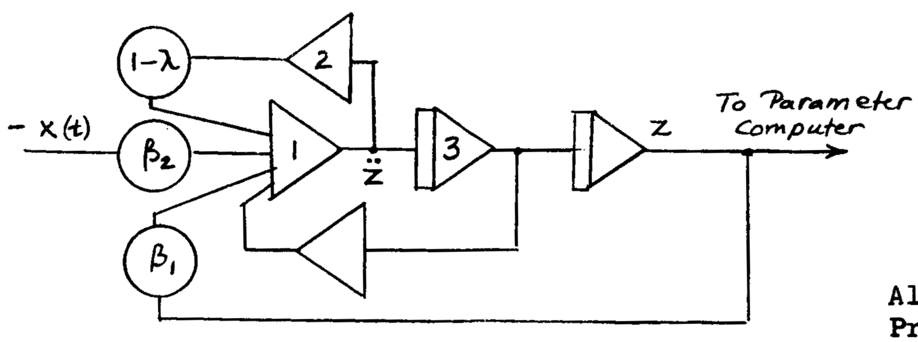


Figure 3

Alternate Computer Programs for Small λ

3. ANALYSIS OF APPROXIMATION ERROR

As in Reference 2 this analysis will be based on the simple case of a purely sinusoidal excitation signal

$$x(t) = C \sin \omega t$$

Using the notation X, Z, Z_0, Z_1, U_λ, E_1, ... for the Laplace transforms of the respective variables we obtain from (1), (2) and (4)

$$\frac{Z}{X} = \frac{\beta_2}{\lambda s^2 + s + \beta_1} = \frac{a}{b\lambda s^2 + bs + 1} \tag{8}$$

$$\frac{Z_0}{X} = \frac{a}{bs + 1} \tag{9}$$

$$\frac{U_\lambda}{X} = - \frac{ab\ s^2}{(b\ s + 1)^2} \tag{10}$$

where $a = \beta_2/\beta_1$, $b = 1/b_\beta$

Hence the approximation error

$$\epsilon_1 = z - z_1 = z - z_0 - \lambda\ u_\lambda \tag{11}$$

is expressed by

$$\frac{E_1}{X} = \frac{a}{b\lambda s^2 + b\ s + 1} - \frac{a}{bs + 1} + \frac{ab\ \lambda\ s^2}{(bs + 1)^2} \tag{12}$$

For convenience of analysis this equation is normalized by the notation

$\nu = \dfrac{1}{a} \dfrac{E_1}{X}$ - normalized error transfer function

$\eta = \dfrac{\omega}{\beta_1}$ - normalized excitation frequency where β_1 is the cutoff frequency of the model transfer function (9)

$\lambda\beta_1 = \frac{\lambda}{b} = \mu$ - normalized small parameter

$\lambda_1\beta_1 = \frac{\lambda_1}{b} = \mu_1$ - normalized approximation of λ obtained by solution of eq. (5)

$\sigma = \frac{s}{\beta_1}$ - normalized Laplace operator

This results in

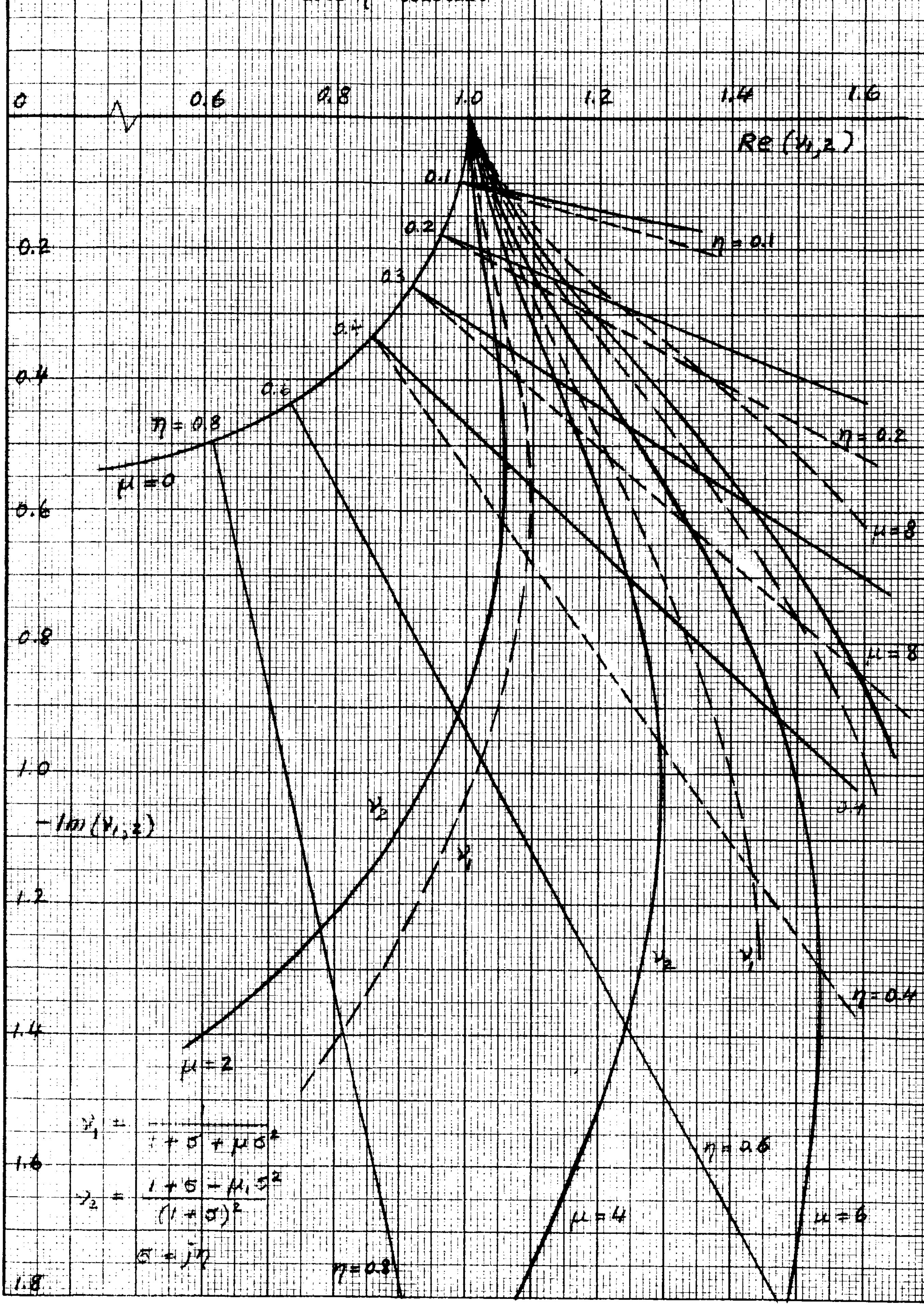
$$\nu = \frac{1}{1 + \sigma + \mu\,\sigma^2} - \frac{1 + \sigma - \mu_1\,\sigma^2}{(1 + \sigma)^2} \tag{13}$$

$$\nu = \nu_1\,(\mu) - \nu_2\,(\mu_1)$$

Frequency response loci of the terms ν_1 and ν_2 are plotted in Figure 4 to show the approximation error ν for several values of the parameter μ. For small frequencies η (0 ... 0.4) and small μ (0 ... 4) the approximation is quite satisfactory but the error increases rapidly as $|\nu_1|$ approaches a peak value near the critical frequency $\eta = 1/\sqrt{\mu}$, i.e., $\omega = \sqrt{\beta_1/\lambda}$.

The best estimate for μ obtainable by solving equation (5) can be determined graphically by means of the loci η = const in Figure 4. The error $\Delta\mu$ in the parameter value μ_1 thus derived is plotted versus frequency η in Figure 5a. The corresponding minimum of $|\nu|$ is plotted in Figure 5b. For frequencies of interest in human pilot models ($\omega \leq 5$ rad/sec) and a typical parameter value β_1 = 50 rad/sec, i.e., for $\eta \leq 0.1$, these results show that λ_1 will be determined with an error of less than 10 percent for $\lambda \leq 0.04$ ($\mu \leq 2$). The output amplitude error of z_1 is less than 1 percent under these conditions. For ω = 10 rad/sec (η = 0.2) the error in λ increases to 30 percent while the amplitude error is still only 1 percent. As the frequency increases, the parameter values μ which permit a satisfactory approximation by this technique decrease sharply, as shown in Figure 5a. The accuracy of the technique is comparable to the analogous approximation of time delay τ discussed in Reference 2.

Figure 4. Frequency Response Loci of ν_1, ν_2 and Loci η = Constant



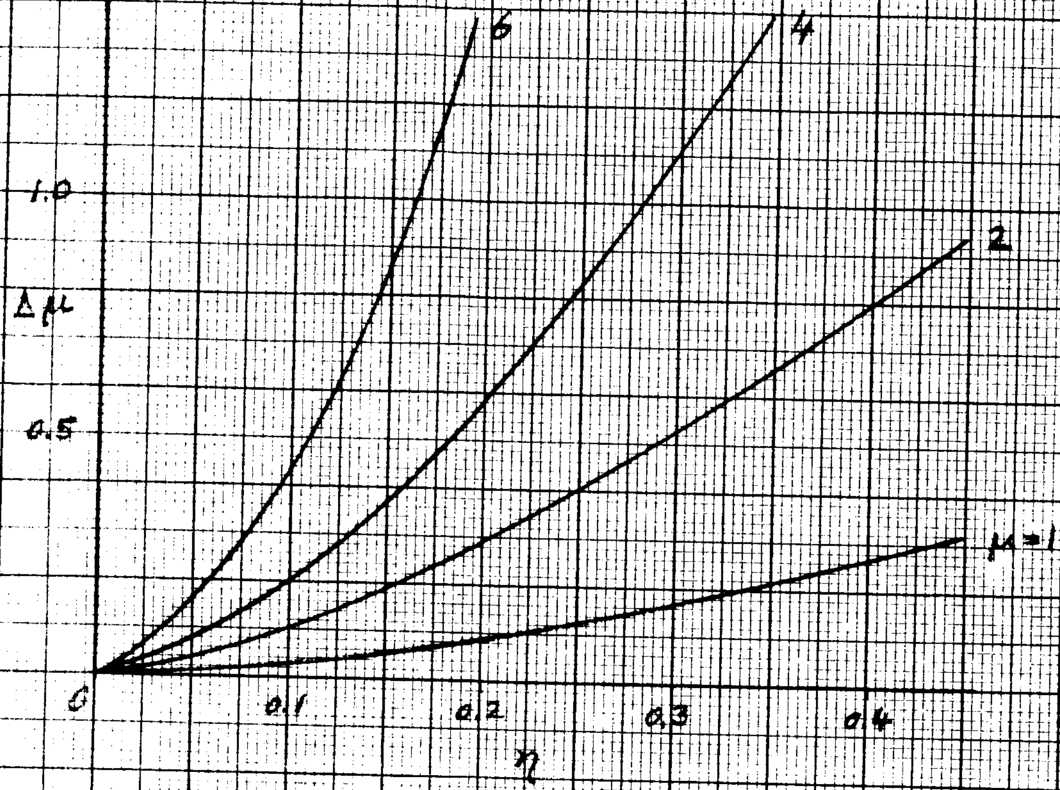


Figure 5a

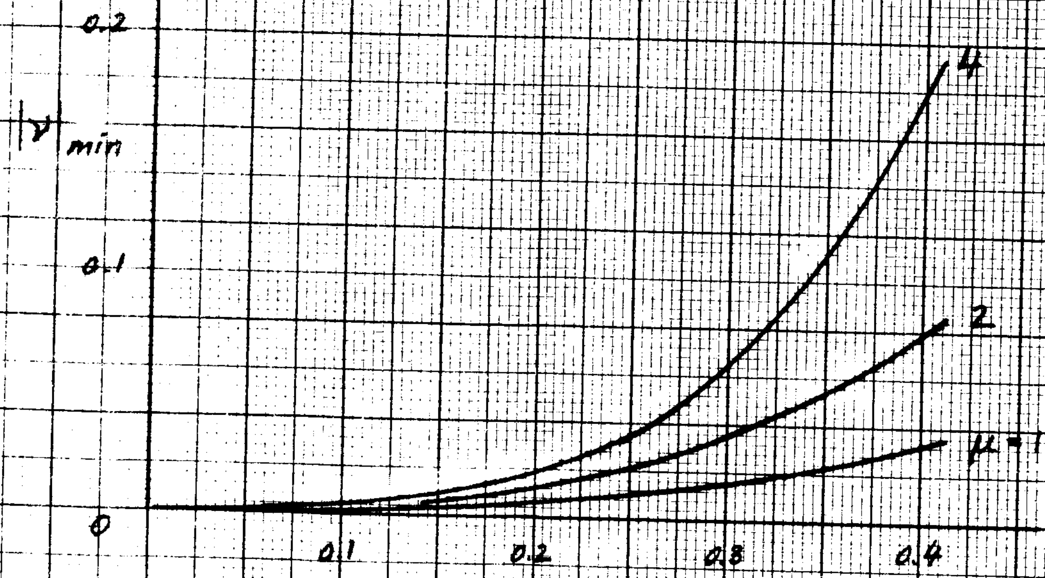


Figure 5b

Figure 5. Parameter Estimation Error $\Delta\mu$ and Amplitude Error $|\nu|_{min}$ Versus Frequency

4. DISCUSSION

The above analysis has shown that small parameter values λ can be approximated with acceptable accuracy in the vicinity of $\lambda = 0$ when the model equation is subject to a low frequency sinusoidal excitation signal. Theoretical considerations concerned with a singular point at $t = 0$ do not cause practical problems since the parameter identification process takes place over a time interval when the effect of initial conditions has subsided. The model parameters reflect the dynamic response of the system under continuous excitation $x(t)$.

If the excitation $x(t)$ is a random signal rather than a sinusoid the results of this simplified analysis can be used to estimate the approximation accuracy for λ for given input bandwidths.

Although this discussion has been restricted to a first order model with a missing second order term the analysis can be readily extended to higher order problems and will probably yield errors of comparable magnitude.

On the basis of the above analytical confirmation of the approximation method experimental evaluations of missing higher order terms will be included under Task 2 of the study program.

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INTEROFFICE CORRESPONDENCE

9350.6-155

TO: Distribution CC: DATE: 3 March 1965

SUBJECT: Modified Parameter Optimization Strategy Using Exact Gradient Components

FROM: H. F. Meissinger
BLDG. R2 ROOM 1086 EXT. 22115

1. Objective

The parameter optimization strategy currently used for identification of human pilot parameters (References 1, 2) is based on approximate gradient techniques for finding the minimum of the error criterion $F = F(\epsilon)$. In this program the definition of gradient components during continuous adjustment of the model parameters is not mathematically rigorous. The error sensitivities $\partial F/\partial \alpha_1$ are defined only when the parameters α_1 remain time-invariant. If the parameters are adjusted very "slowly," i.e., at rates much below the frequency characteristics of the system in question, an acceptable approximation of the gradient is obtained, suitable for a steepest descent optimization strategy (Reference 3). Inability to define and adequately approximate the gradient causes problems when the adjustment rate is significantly increased to obtain rapid convergence of the parameters to their optimum values.

A second but equally important problem stems from the fact that in the present adjustment strategy the parameter adjustments do not feed back instantaneously to the model output, the point at which the model matching error ϵ is detected and the error criterion $F(\epsilon)$ is formed. The parameter adjustments $\Delta\alpha_1$ pass through several stages of integration before affecting the error criterion. In a second order model, e.g., the parameter variations are integrated twice before producing the corresponding variation ΔF as illustrated in Figure 1. The loop is closed through a third integrator.

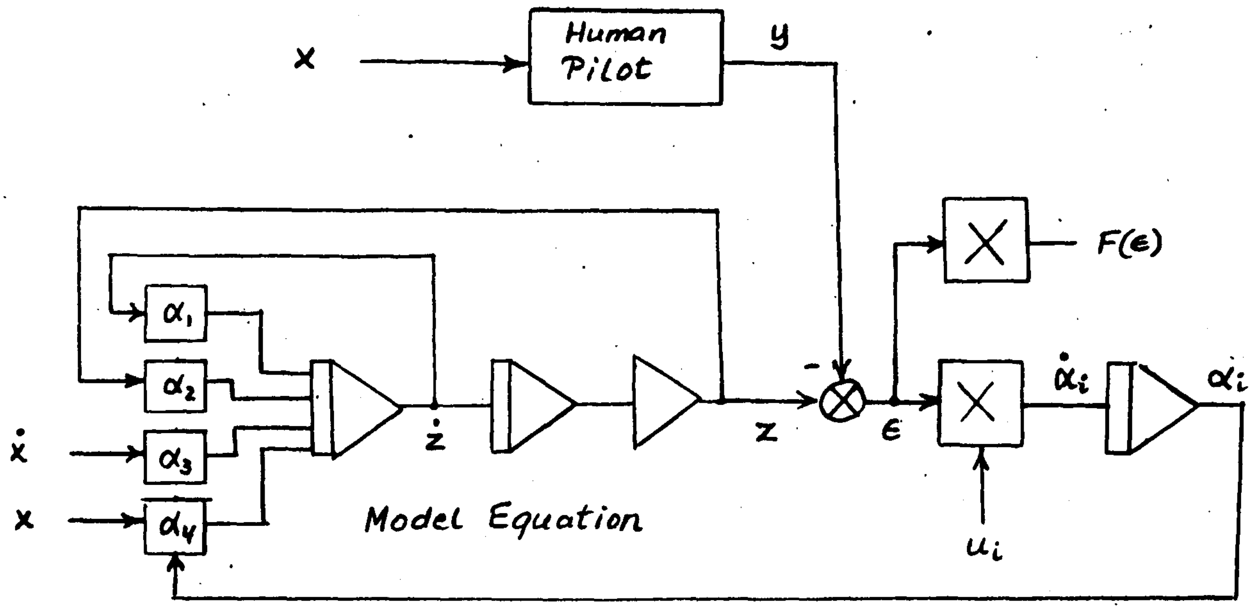

Figure 1. Current Parameter Adjustment Program with Time-Lag in Formation of F(ε)

This can cause stability problems in the parameter adjustment loops, unless the adjustment circuits are designed with care to have appropriate gain and phase lead characteristics. In view of the highly nonlinear character of this feedback system the analysis of stability and convergence characteristics is difficult and has not been performed in the general n-parameter case. The choice of the adjustment gain and stabilization network on the computer is usually derived on an experimental basis and must be adapted to varying operating conditions of the system being modeled.

The objective of the modified adjustment strategy is to minimize these problems inherent in the present model matching program. The new program eliminates the problem of inexact mathematical definition of gradient components arising from rapid parameter adjustment, since the error criterion F(ε) is no longer a "functional" of the parameter variation time history but is defined as an algebraic (linear) function of the parameter increments $\Delta\alpha_i$. The gradient

$$\bar{\nabla}F = \left(\frac{\partial F}{\partial\Delta\alpha_1}, \frac{\partial F}{\partial\Delta\alpha_2}, \dots \frac{\partial F}{\partial\Delta\alpha_n}\right) \tag{1}$$

therefore is always rigorously defined during the adjustment process. The error criterion, furthermore, responds instantaneously to the α_i-adjustment which minimizes the stabilization problem of the adjustment loop. As a consequence the computer equipment and programming requirements and the experimental effort in optimizing the gain and phase characteristics of the adjustment circuitry are alleviated. Also the program is more amenable to analysis of stability and convergence characteristics.

2. Modified Optimization Program

The new optimization strategy is formulated as follows. The model equation and the sensitivity equations are solved at the original operating point $\bar{\alpha}_0$ in the parameter space* yielding results z_0 and u_0. This solution is imbedded in a family of solutions obtainable for different fixed parameter settings (see Figure 2). This family can be approximated by extrapolation as follows:

$$z_1 = z_0 + u_1\,\Delta\alpha_1 + u_2\,\Delta\alpha_2 + \dots \tag{2}$$

Thus the parameter variations $\Delta\bar{\alpha}$ obtained by the steepest descent method are used only to extrapolate from the "parent solution" z_0 to members of the family of solutions in which z_0 is imbedded. This eliminates the mathematical difficulty of characterizing z as a function of the adjustable parameters $\bar{\alpha}$ and defining the partial derivatives $\partial z/\partial\alpha_i$. The partial derivatives $\partial z/\partial\alpha$ and hence $\partial F/\partial\alpha$ are rigorously defined at all times.

* Vector notation $\bar{\alpha} = (\alpha_1, \alpha_2, \dots)$

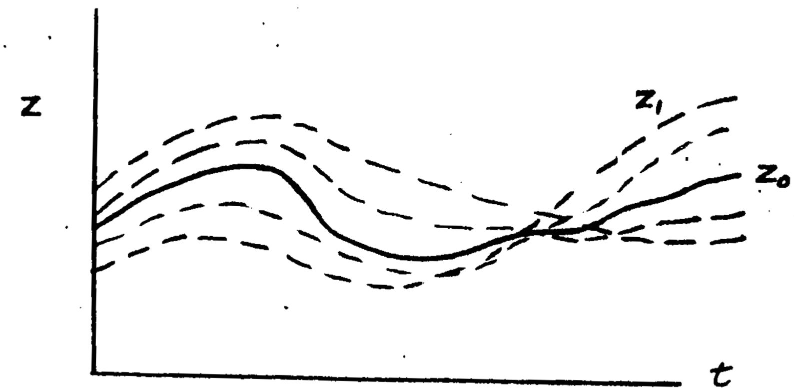

Figure 2. Extrapolated Solutions z_1 in Vicinity of Parent Solution z_0

In addition, the variation of the instantaneous model matching error with $\bar{\alpha}$ variation does not contain dynamic time lags so that instantaneous improvement of the error criterion can be obtained in the optimization process. Mathematically, the only approximation error to contend with is the extrapolation error as one departs further and further from the original operating point $\bar{\alpha}_0$. This means that several iterations may be required to attain the optimum, depending on the amount of total parameter correction. Each iteration proceeds to the best parameter values attainable within the family of extrapolated solutions in the vicinity of $z_0^{(1)}$, $z_0^{(2)}$ ... etc. It is anticipated that only few iterative steps are required in most cases of interest.

The new strategy* is inherently related to the "open loop" technique described in Reference 4 but focuses more directly on the imbedding of the parent solution within adjacent solutions.

3. Discussion

To explore the modified optimization program further it will be necessary to perform an experimental model matching study and to compare this with results of the present adjustment strategy. An indication of the feasibility and usefulness of the method can be gained by the preliminary "open loop" optimization experiments described in Reference 4 (see Figure 4). In this case two parameters of a second order system

* Illustrated in Figure 3.

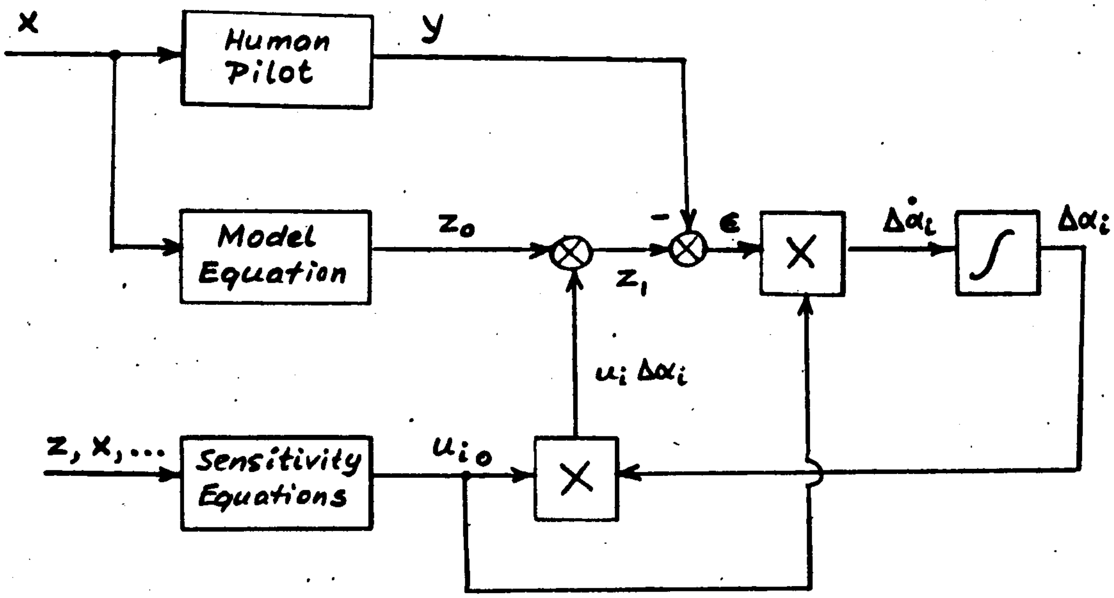

Figure 3. Modified Parameter Adjustment Program

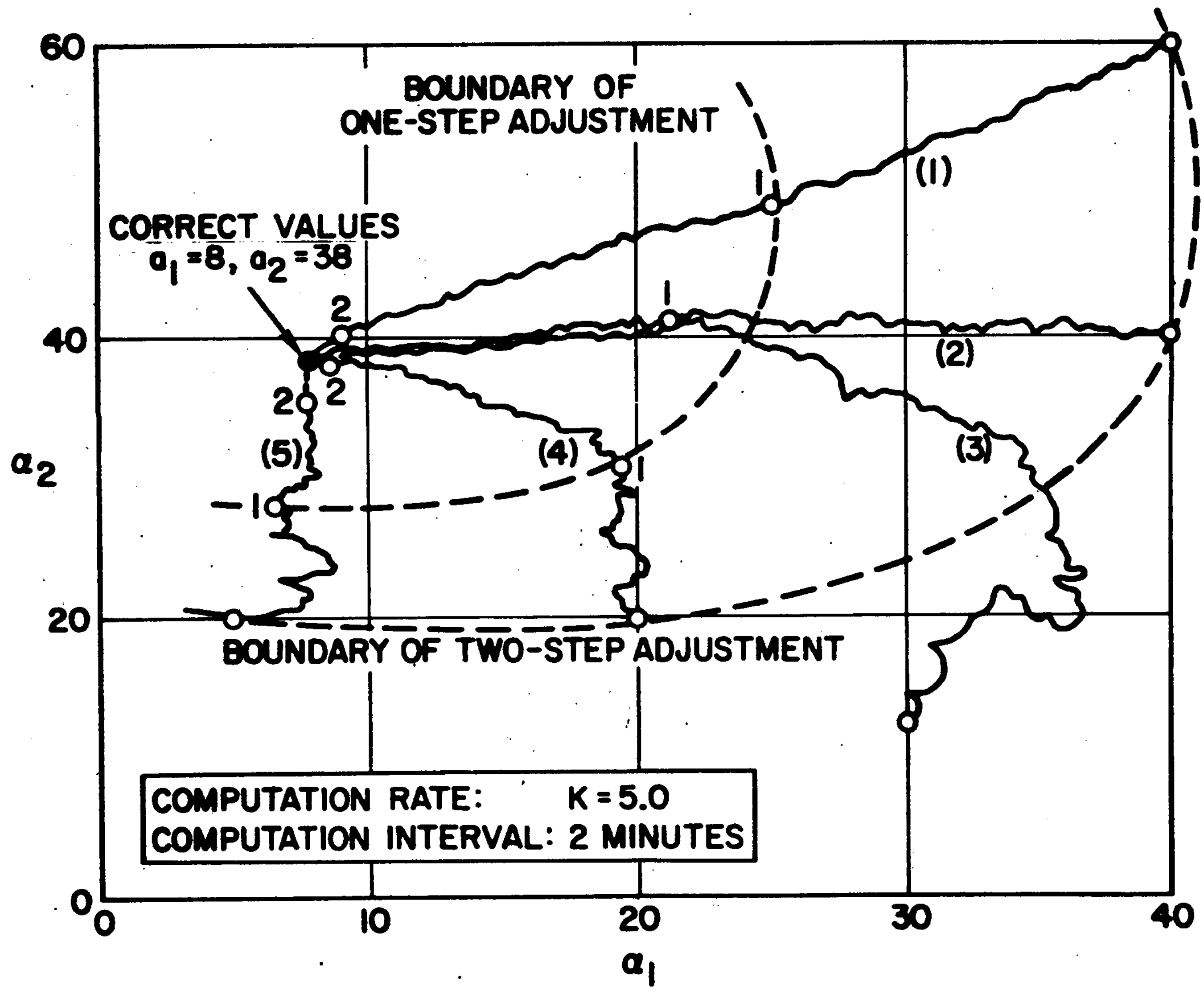

FIGURE 4. OPTIMIZATION TRAJECTORIES IN a_1, a_2 PLANE

were adjusted to their (known) correct values in 1, 2, or 3 steps of iteration, depending on the initial error. It was shown that there exists a sizable area in the parameter space in the vicinity of the optimum in which a one-step optimization is sufficient. Extension to more complex linear and nonlinear models is intended.

Equipment savings can be realized by the new strategy through elimination of the lead compensation term $q\dot{\epsilon}$ in the criterion function, which also eliminates the terms $q\dot{u}_i$ in the adjustment equations used previously,

$$\Delta\dot{\alpha}_i = -K(\epsilon + q\dot{\epsilon}) \quad (u_i + q\dot{u}_i) \qquad (3)$$

This simplification has the desirable effect of eliminating differentiation circuits for $\dot{\epsilon}$ and some of the $\dot{u}_i$ in the computer program.

In cases where repeated iterations are required the program calls for automatic sequencing equipment to hold, reset, and operate the adjustment circuits. Since adjustment rates can be stepped up it is anticipated that the overall time for parameter determination can be reduced even with several iterations required. Under favorable conditions when iterations are unnecessary due to good initial estimates of the parameter settings, all multipliers for parameter adjustment in the model equation and sensitivity equations can be eliminated along with the sequencing equipment, and the time savings will be very significant.

H. F. Meissinger

H. F. Meissinger

HFM:js
Distribution

G. A. Bekey
W. A. Finley
R. E. Rose
L. G. Summers
E. P. Todosiev

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1. "A Study of Model Matching Techniques for the Determination of Parameters in Human Pilot Models," by G. A. Gekey, H. F. Meissinger, and R. E. Rose. STL No. 8426-6006-RU000, dated 2 May 1964. (Published under NASA Contractor Series CR-143, January 1965.)

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INTEROFFICE CORRESPONDENCE

9352.2-5

TO: Distribution

CC:

DATE: 9 March 1965

SUBJECT: Evaluation of Display Sensitivity by Human Operator Models

FROM: L. G. Summers

BLDG. R2 ROOM 1186-f EXT. 22 871

This memorandum is a description of a preliminary experiment utilizing the model matching technique for the evaluation of display characteristics. It was conducted on Manual Control and Visual Perception Independent Research funds for calendar years 1964 and 1965. This experiment provides support for the methodology proposed in Proposal 3984.00 "Cockpit Display Evaluation by Means of Human Pilot Model", 5 August 1964 to NASA/Edwards. It also provides a practical application for the model matching techniques being developed under Contract NAS 1-4419.

L G Summers

L. G. Summers

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EVALUATION OF DISPLAY SENSITIVITY BY HUMAN OPERATOR MODELS

INTRODUCTION AND SUMMARY

The objective of this preliminary study was to determine the effect of display characteristics upon the parameters of the human operator's dynamic response model. The initial hypothesis was that changes in display characteristics would affect the operator's model without affecting the performance error. If it could be shown that parameter values of the model do change, implying a variation in operator effectiveness, this analysis would be useful for display evaluation.

To test the above hypothesis, display sensitivity was used as a change in display mode. Three conditions were initially planned, 1) viewing a CRT through an eyepiece so that the field-of-view would be 60° and one degree of signal deflection would subtend a visual angle of one degree at the eye, 2) viewing a CRT at a distance of 28 inches where the display sensitivity was the same as that of the Lear-Siegler Attitude Indicator, and 3) utilization of the Lear-Siegler Attitude Indicator as the display viewed from a distance of 28 inches. The last condition was deleted since it was realized the dynamic response characteristics of the display would have to be analyzed before application of model matching.

In analysis of root-mean-square error in tracking performance, no significant difference was found between the two display sensitivities. The data were then analyzed by model matching utilizing a first-order model format. A trend was shown in the operator gain parameter which was statistically significant at the 10% level. With the less sensitive display the operator lowered his gain. This indicates that the operator lowers his crossover frequency implying that the task is more difficult.

The conclusion of this experiment is that the analysis of operator dynamics in relation to display dynamics is not only a novel but a valid display evaluation technique. This study also implies that the differences in display effectiveness may be attributed to differences in the dynamics of the display and not display formats. The next step in such a program would be to determine the effect of display format and redundant cues upon the operator's dynamics.

METHOD

The task utilized to test the hypothesis was a compensatory tracking task with a random forcing function input. A block diagram of the task arrangement is shown in Figure 1. The

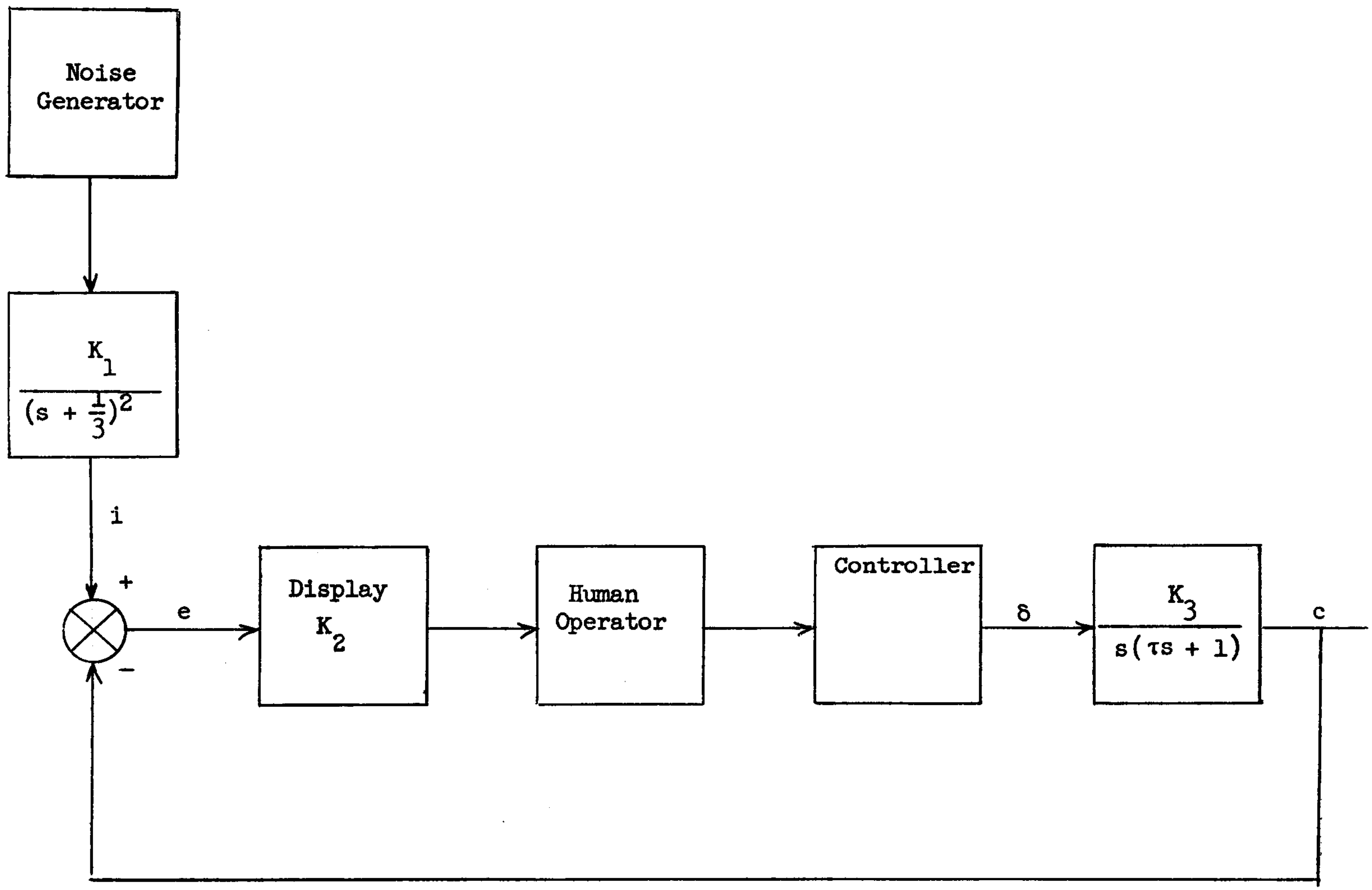

Figure 1

Block Diagram of Tracking Task Arrangement

The forcing function was generated by a random noise generator. This signal was filtered by a third order filter with a cutoff frequency of 0.33 rad/sec. The RMS amplitude of the input signal averaged 15.1 °/sec. The task dynamics was second order with a single lag time constant of 0.5 secs and a gain of 1.09 rad/sec per radian of stick deflection. The display element was a horizon line and tracking was in the vertical axis corresponding to pitch deflection. A fingertip controller was used with the shaft of the controller in a horizontal position. Maximum displacement of the controller was $\pm 27^{\circ}$. Upward movement of the controller corresponded to pitching up, therefore the horizon would drop, representative of an inside-out display.

For the out-of-the-window display sensitivity a 5" CRT was viewed through an eyepiece. The eyepiece allowed the CRT face to appear at infinity and cover a field of view of 60°. One degree of pitch deflection was represented by one degree deflection of the horizon line viewed through the eyepiece. Hereafter, this sensitivity will be referred to as a sensitivity of 1. The attitude indicator sensitivity was obtained by measurement of the visual angle of a given horizon displacement from a distance of 28". This sensitivity was 0.071 degrees per degree of pitch deflection. Therefore the attitude indicator was 14 times less sensitive than out-of-the-window viewing.

Four experimental subjects were used in a full factorial design. Each subject was given 20 replications of the 1 sensitivity in two sessions. The last four runs of the second session were used for data analysis and the last two were recorded on FM tape for analysis by model matching. After the subjects had received the 1 sensitivity condition they received 10 replications of the 0.071 sensitivity condition in two sessions. Again the last four runs were used in the data analysis and the last two were recorded on FM tape.

The subjects were instructed to minimize the error. They were given feedback on their performance error between replications. The length of each run was 2-1/2 minutes.

The model parameters were obtained by the parameter optimization technique as developed by Bekey, Meissinger and Rose (Reference 1). The model format was a first order differential equation as follows,

$$\dot{z} + \beta_1 z = \beta_2 \dot{x} + \beta_3 x$$

where z is the model output and x is the error input. This equation can be rewritten in the transfer function form

$$z/x = \frac{\frac{\beta_3}{\beta_1}\left[\frac{\beta_2}{\beta_3} s + 1\right]}{\frac{1}{\beta_1} s + 1} \quad \text{or,}$$

$$= \frac{K_p \ (T_1 \ s + 1)}{(T_2 \ s + 1 \ )}$$

where K_p is the operator's gain, T_1 is the lead time constant, and T_2 is the lag time constant. The modified absolute value error criterion was used for the parameter optimization (Reference 2).

RESULTS

The root-mean-square error was used as the performance criterion for the experiment. The average values across four replications for each condition and subject are given in Table 1. In order to test the significance of the data an analysis of variance test was administered. The design for the test was a treatment by subjects with within-cell [illegible]plication. This analysis is shown in Table 2 for the four replications. The only significant variation in the scores is between subjects. The 10% significant interaction between display sensitivity and subjects was probably due to one subject not having sufficient training.

Another analysis of variance test was conducted for only the two replications that were recorded on FM tape. This test would allow comparing the significance of the error scores with the parameter values from the model matching. This analysis is shown in Table 3.

The average value of each parameter over a 120 second period was determined from the model matching results. A power match to determine how much of the model accounted for stick output was calculated by

$$1 - \frac{S\int e^2\, dt}{S\int y^2 dt},$$

where e is the matching error and y is the stick output. This was calcuated for selected [illegible]hs where the parameter values were held constant at their average value. This match ranged between .70 to .85 of the stick output. The parameter values are given in Table 4. An analysis of variance was conducted for K_p, T_1, and T_2. This analysis is presented in

Table 5. This shows that for the operator's gain there was a significant difference between sensitivities at the 10% level. For the lag time constant the interaction term was significant at the 10% level. In observing the parameter values in Table 4 there is a lower operator gain associated with the 0.071 display sensitivity. The interaction in the lag time constant analysis was probably due to insufficient training in one subject.

TABLE 1.

RMS Error Scores Averaged over Four Replications for Each Subject and Display Sensitivity

Display Sensitivity	Subject 1	Subject 2	Subject 3	Subject 4
1	30.1	18.2	27.2	22.3
0.071	27.2	21.3	27.8	21.8

TABLE 2.

Analysis of Variance for Four Replications of RMS Errors

	S S	D of F	M S	F
Display Sensitivity	2	1	2	0
Subjects	43284	3	14426	11.7**
Interaction	3691	3	1230	2.54*
w	11642	24	485	

** Significant at 0.05 level

* Significant at 0.10 level

TABLE 3.

Analysis of Variance for Two Replications of RMS Error Score

	S S	D of F	M S	F
Display Sensitivity	121	1	121	0.15
Subjects	26826	3	8942	11.5**
Interaction	2327	3	776	1.28
W	4855	8	607	

TABLE 4.

Average Parameters for Each Subject and Display Sensitivity

	K_p Operator Gain (rad of stick/rad of θ)	T_1 Lead Time Constant	T_2 Lag Time Constant
1.00 Sensitivity			
S 1	4.42	0.099	0.049
S 2	2.62	0.039	0.032
S 3	3.93	0.106	0.032
S 4	1.78	0.008	0.024
0.071 Sensitivity			
S 1	2.61	-0.022	0.042
S 2	2.37	0.014	0.036
S 3	2.34	0.108	0.036
S 4	1.41	0.096	0.034

** Significant at 0.05 level

TABLE 5

Analysis of Variance of the Parameter Values of the First Order Model

Source	Sum of Squares	Degrees of Freedom	Mean Square	F Ratio
K_p (Operator Gain)				
Sensitivity	40602	1	40602	6.201*
Subjects	84933	3	28311	4.324
Interaction	19640	3	6547	.332
w	157689	8	19711	
T_1 (Lead Time Constant)				
Sensitivity	341	1	341	0.049
Subjects	13621	3	4540	0.653
Interaction	20845	3	6948	0.967
w	57505	8	7188	
T_2 (Lag Time Constant)				
Sensitivity	39	1	39	0.70
Subjects	590	3	197	3.54
Interaction	167	3	56	3.84*
w	116	8	15	

* Significant at 0.10 level

CONCLUSIONS

The display gain can be reinterpreted as a change in the control system gain along with an equal change in the forcing function input. If the control system gain is lowered it would be expected that the operator would raise his gain to compensate for it. However, in this circumstance, where the forcing function input's gain is decreased by the same amount, the operator decreases his gain. This indicates that he is operating at a lower crossover frequency with the less sensitive display.

An explanation for the operator lowering his gain with the less sensitive display is that the task is more difficult. This difficulty would be reflected in the RMS error if the bandwidth of the input forcing function had been larger. Interpretation in terms of information theory would be that the information transfer rate is lowered with the less sensitive display.If the operator's task is separated into perceptual, central and motor processing which are serial then the lowering of the transfer rate has to occur in only one of these processes. The most obvious is the perceptual which can be explained by a visual displacement threshold, i.e. the capability of the operator to perceive a displacement of the horizon line from the center line of the CRT. If it is assumed that the operator acts primarily on displacement and if he has an absolute displacement threshold for the display format utilized in this experiment, then he would act upon lower angular displacements in the display with the larger gain and thereby increase his gain or information transfer rate. This theory could be validated if a linear model with a nonlinear threshold term was utilized in the model matching instead of first order linear model. It would have to be determined if the threshold value and/or linear gain, would remain constant or vary.

In either case the results of this preliminary study imply that determination of human operator's models for analyzing display characteristics will provide quantitative data from which display design criteria can be obtained.

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